Comment on "Dynamic range of nanotube- and nanowire-based electromechanical systems" [Appl. Phys. Lett. 86, 223105 (2005)]

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The field of nanoelectromechanical systems (NEMS) has attracted great interest in the last decade and a variety of different applications of NEMS (such as ultrasensitive force and mass detectors) have been proposed. Very recently, an interesting work on the dynamic range of nanotube- and nanowire-based NEMS has been presented.[1]

The aim of this comment is to emphasize quantum effects in the dynamic range (DR) of nanoscaled resonators. We demonstrate that the zero-point energy contribution starts to play a significant role near the temperature and size regimes considered in Ref. 1.

A crucial quantity in the derivation of the DR [see Eq. (8) in Ref. 1] is the thermomechanical noise, expressed by the spectral density of the displacement noise (on resonance ω_0) given by [see Eq. (7) in Ref. 1],

$$S_x = 4k_B T \frac{Q}{m\omega_0^3}, \tag{1}$$

which is based on the classical equipartition law ($k_B T \gg \hbar\omega_0$) and on the classical fluctuation-dissipation theorem (CFDT). However, since the systems discussed in Ref. 1 will be driven at low temperatures (see the ratios of $\hbar\omega_0/(k_B T)$ in Table I) quantum fluctuations should be taken into account in the thermomechanical noise. The most direct way to include the quantum noise is to substitute the thermal energy $k_B T$ by the thermally averaged quantum energy including the zero-point energy contribution (following Callen and Welton[2]). Employing the quantum fluctuation-dissipation theorem (QFDT), the expression for the spectral density of the displacement noise (again on resonance ω_0) is thus given by (see Ref. 3 and references therein)

$$S_x = 2\hbar\omega_0 \coth\left(\frac{\hbar\omega_0}{2k_B T}\right)\frac{Q}{m\omega_0^3}. \tag{2}$$

In the high-temperature limit ($k_B T \gg \hbar\omega_0$), we recover Eq. (1), corresponding to the results of Nyquist and Johnson.[4,5] Substituting Eq. (2) into the expression for the DR [Eq. (8) in Ref. 1] leads to the modified DR based on QFDT. A replot of Fig. 3 in Ref. 1 is shown in Fig. 1, where the DR based on the CFDT and the QFDT are plotted as a function of the resonator length. Note the significant discrepancy between the classical and the quantum DR in temperature and length regimes accessible by the experiment. The numerical data shown in Fig. 1 and in Table I have been computed using Eq. (9) and the parameters given in Table 1

TABLE I. Ratio $\hbar\omega_0/(k_B T)$ for several doubly clamped resonators at 4 K. The results for the single-walled carbon nanotube (SWNT) are calculated with the model of a solid ˜ cylinder (as in Ref. 1) and, alternatively, with the model of a hollow cylinder (see results in brackets): diameter $d=1.4$ nm, section area $A=\pi dt$, moment of inertia $I=d^3 t/8$, and the layer spacing for graphite $t=0.34$ nm.

$\hbar\omega_0/(k_B T)$	$l=200$ nm	$l=100$ nm	$l=50$ nm	$l=25$ nm
SWNT	0.008 (0.024)	0.03 (0.098)	0.13 (0.39)	0.55 (1.57)
MWNT	0.06	0.25	0.99	3.96
Pt nanowire	0.03	0.13	0.53	2.11
SiC resonator	0.57	2.30	9.21	36.8

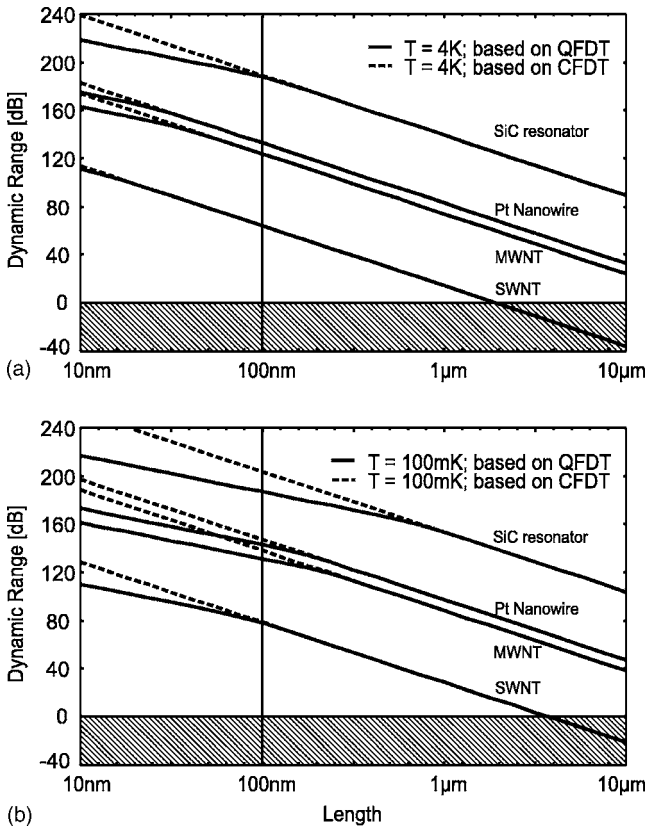

FIG. 1. Dynamic range as a function of resonator length at (a) 4 K and (b) 100 mK for several doubly clamped resonators based on the classical and quantum fluctuation-dissipation theorem (C/QFDT). Note the significant discrepancy between the DR based on CFDT vs QFDT and the expected temperature independence of the QFDT results for short resonator length.

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of Ref. 1. However, to reproduce Fig. 3 of Ref. 1 we chose the diameter of the multiwalled carbon nanotube (MWNT) as $d=10$ nm and the Q-factor of the SiC beam as $Q=4500$ (as in Ref. 18 of Ref. 1).

In conclusion, we have shown that the quantum fluctuation-dissipation relation poses limits on the dynamics of nanoscaled mechanical resonators and cannot be neglected in a description of NEMS at low temperatures.

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