Supplemental material for
"Probing decoherence through Fano resonances"

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COMPLEX q-TRAJECTORIES: THE CASE OF DEPHASING

Following previous analysis [1] based on the Büttiker dephasing probe model, we show in the main text of our article (referred to as Ref. [I]) that for this specific model the Fano q-parameter follows a circular trajectory in the complex plane. It is now instructive to inquire whether this result can also be found for a more general scenario of dephasing. One such generic approach is the ensemble average over a random phase ϕ between the resonant ($\propto z_r$) and the background amplitude t_d,

$$t = t_d + \frac{z_r}{i+\varepsilon}\, e^{i\phi} \,,$$

where the reduced wavenumber $\varepsilon = (k - k_{\text{res}})/(\Gamma/2)$, see Eq. (3) of [I]. With the random phase ϕ featuring a zero mean and a standard deviation σ the interference term in the ensemble average of the total transmission $\langle |t|^2 \rangle_\phi$ will be gradually suppressed for increasing σ. This behavior can be conveniently described by a prefactor $(1-\chi) \in [0, 1]$, containing the dephasing strength $\chi(\sigma)$,

$$\begin{aligned} \langle |t|^2 \rangle_\phi &= |t_d|^2 + \frac{|z_r|^2}{1+\varepsilon^2} + (1-\chi)\, 2\text{Re}\left(t_d^* \frac{z_r}{i+\varepsilon} \right) \\ &= |t_d|^2 \, \frac{|z_r/t_d|^2 + (1-\chi)\, 2\text{Re}\left[\frac{z_r}{t_d}(-i+\varepsilon) \right] + (1+\varepsilon^2)}{1+\varepsilon^2} \,. \end{aligned}$$

The limit of complete dephasing ($\chi \to 1$) corresponds to the incoherent addition of resonant and background contribution. With the real value of the Fano parameter q_0 in the absence of decoherence given as $q_0 = i + z_r/t_d$ [1], we further obtain,

$$\langle |t|^2 \rangle_\phi = |t_d|^2 \, \frac{\left[\varepsilon + (1-\chi) q_0\right]^2 - (1-\chi)^2 q_0^2 + q_0^2 + 2\chi}{1+\varepsilon^2} \,.$$

Comparing this expression with the general form of a Fano resonance as in Eq. (6) of [I] reveals the evolution of the complex Fano parameter $q(\chi)$ as a function of the decoherence strength χ,

$$\begin{aligned} \text{Re}[q(\chi)] &= (1-\chi)\, q_0 \,, \\ \text{Im}[q(\chi)]^2 &= 2\chi + q_0^2\, (2\chi - \chi^2) \,. \end{aligned}$$

Eliminating χ from the above two equations finally yields,

$$\text{Im}[q(\chi)]^2 + \{\text{Re}[q(\chi)] + 1/q_0\}^2 = 2 + \frac{1}{q_0^2} + q_0^2 \,,$$

which describes a circle in the complex plane with radius $r_0 = \sqrt{2 + 1/q_0^2 + q_0^2}$, centered at $x_0 = -1/q_0$ on the real axis. Note that in the limit of complete dephasing ($\chi \to 1$) the above trajectory $q(\chi)$ converges to a value on the imaginary axis which, in contrast to the result obtained with the Büttiker dephasing probe model [1], is not necessarily given by $q = i$.

We thus arrive at the interesting conclusion that different models of dephasing may yield a similar circular behavior of $q(\chi)$ where, however, the radius of the circular arc and its corresponding end point depend on the specific dephasing scenario.

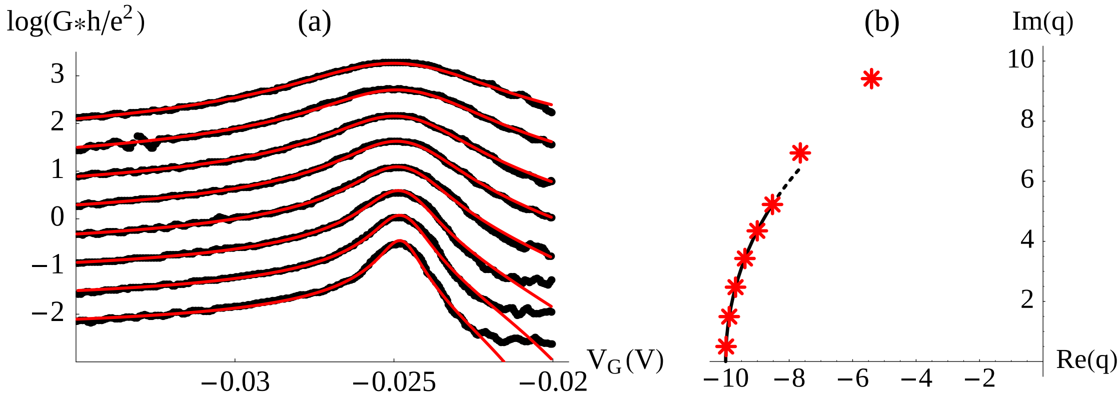

Fig. E1: (a) Voltage dependence of the conductance through a resonant quantum dot as measured in the experiment by Zacharia et al. [2]. The black dots show the values extracted from [2] in steps of 100 mK between 100mK (lowest curve) and 800mK (highest curve). For better visibility the data sets with $T > 100$ mK are each multiplied by a factor of $\sqrt{10}$ from one temperature value to the next. The red curves show the analytical curves resulting from restricted parameter fits (see text) with the corresponding complex q-values being displayed in (b) by the red asterisk symbols. The black curve in (b) shows a circular arc as prescribed for the dephasing model of decoherence.

DETAILS ON THE EMPLOYED FITTING PROCEDURES

When extracting the complex value of q from a resonance profile, the imaginary part of q is highly sensitive to the minimum value of the profile. General automated fitting routines, however, do typically not account for this specific dependence appropriately. To overcome this difficulty we used the minimum, maximum, and an intermediate point of the resonance as interpolation points in our procedure. Furthermore, we took advantage of the knowledge that the minimum and maximum points are extreme values of the Fano resonance, resulting in altogether five equations. Their solution yields the resonance amplitude, the resonance position k_{res}, the resonance width Γ and the real and imaginary part of q. All the q-values in Fig. 3 of [I] corresponding to the microwave experiment and to the numerical dephasing data were extracted in this way. Resonances with a non-uniform background were excluded from our analysis.

As outlined in the main text, we also tested our predictions against experimental data previously published in [2]. In that publication the resonant conductance through a quantum dot was studied as a function of temperature T (100mK $\leq T \leq$ 800mK). The corresponding experimental data extracted directly from [2] are shown in Fig. E1(a) (black dots), right above. Unfortunately, for all the resonance curves provided in [2] the background transmission is very small, such that the corresponding Fano resonance lineshapes in the conductance are very near the symmetric Breit-Wigner limit (with $|q| \gg 1$). In this limit we find rigorous multi-parameter fits with a complex-valued q-parameter as performed on the more asymmetric Fano resonances (with $|q| \lesssim 1$ as in the microwave data) to be unfeasible. This is because for $|q| \gg 1$ the symmetry-restoring effect of decoherence is hard to quantify. To test if the quantum dot data can be described by our theoretical predictions, we instead performed a consistency check whether a trajectory $q(\chi)$ can be found that features good agreement with both the experimental data and a circular form of $q(\chi)$. For this purpose we carried out restricted parameter fits which, in line with [2], were performed on a logarithmic conductance scale with thermal broadening being included separately and the data for the fit being restricted to the range where the conductance is at least twice as large as the measured conductance minimum (to reduce the influence of neighboring peaks). The best curves which we found in this way are shown in Fig. E1(a) (red lines) with the corresponding q-values shown in Fig. E1(b) (red symbols) and in Fig. 3(b) of [I] (after rescaling to the unit circle). The overall very good agreement demonstrates that the arc-like behavior of $q(\chi)$ can very well describe the experimental data. To cross-check this result we also mapped the measurement data on a linear $q(\chi)$-trajectory (not shown) as prescribed by the dissipation-dominated decoherence in Eq. (5) of [I] and found much larger discrepancies. We hope future quantum transport experiments will make more asymmetric Fano resonances (with $|q| \lesssim 1$) available for rigorous analysis.

[1] A. A. Clerk, X. Waintal, and P. W. Brouwer, Phys. Rev. Lett. **86**, 4636 (2001).
[2] I. G. Zacharia, D. Goldhaber-Gordon, G. Granger *et al.*, Phys. Rev. B **64**, 155311 (2001).