Supplemental Material to: Spatiotemporal Control of Light Transmission through a Multimode Fiber with Strong Mode Coupling

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A. WEAK VS. STRONG MODE COUPLING

Principal modes (PMs) in multimode fibers (MMFs) with weak mode coupling have been well studied in [1]. In our experiment, we intentionally applied stress to the fiber to achieve strong mode coupling. To illustrate the fundamental difference between PMs in the weak and strong mode coupling regimes, we conducted a numerical simulation using the same model as described in the main text. In this model, the fiber is divided into many short segments, in each of which light propagates as in a perfect fiber without mode coupling. In between adjacent segments, the guided modes are randomly coupled. The scattering is simulated by a unitary random matrix. The magnitude of the matrix elements decays away from the diagonal. The decay rate determines the degree of mode coupling, a slower decay leads to stronger mode coupling.

When the mode coupling is weak, each mode is coupled only to the ones with similar propagation constants. Thus the transmission matrix (in the mode basis) is nearly diagonal [Fig. S1(a)]. As shown in Fig. S1(c), the output field pattern of a PM bears similarity to that of an eigenmode of the perfect fiber without mode mixing. The modal decomposition of the PM in Fig. S1(e) reveals that the PM is composed of a few modes with similar propagation constant. In the strong coupling regime, by contrast, all the modes are mixed. Both diagonal and off-diagonal elements of the transmission matrix have comparable amplitude [Fig. S1(b)]. The field profile of a PM looks like a speckle pattern [Fig. S1(d)], and it is composed of all the modes [Fig. S1(f)].

To further illustrate the difference between weak and strong coupling regimes, let us consider an example in which light is injected into a few modes with slightly different propagation constants. In the weak coupling regime, the light will remain mostly in these modes when propagating through the fiber. The variation of the output field pattern with input frequency reveals 'mode beating', namely, the spectral correlation function for the output field pattern exhibits oscillations with input frequency [Fig. S1(g)] very similar to those observed in [1]. However, if mode coupling is strong, light will quickly spread to all the modes. The output field pattern decorrelates with frequency detuning, i.e., the spectral correlation function decays instead of oscillating with input frequency [Fig. S1(h)].

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B. EFFECT OF POLARIZATION

In our experiment, the transmission matrix was measured for a single linear polarization of input and output light. Since the polarization is scrambled in the MMF, some of the input light is converted to the other polarization and thus not measured at the output. Consequently, the transmission matrix is non-unitary even without intrinsic loss in the fiber, and it is part of the full transmission matrix for both polarizations. Nevertheless, we can still obtain the time-delay matrix for one polarization from the partial transmission matrix. Its eigenstate gives a linearly polarized input wavefront and generates an output field whose one polarization component has a frequency-invariant spatial profile.

To investigate the effect of polarization on the PM, we performed numerical simulation using the concatenated waveguide model [2]. The two polarizations are assumed to have the same propagation constant. We obtained the time-delay matrix Q from the full transmission matrix (for both polarizations) and the partial transmission matrix (for one polarization). Then we calculated the eigenstates and compared their spectral and temporal properties. Our numerical results confirm that the PMs for one polarization exhibit the same characteristic as the PMs for both polarizations. Their spectral correlation widths overlap, as shown in Fig. S2(a,b), regardless of the mode-dependent loss.

C. SPECTRAL CORRELATION FUNCTION

The spectral correlation function is defined as

$$C(\Delta\omega) \equiv |\hat{\psi}(\omega_0 + \Delta\omega) \cdot \hat{\psi}(\omega_0)| = \cos[\theta(\Delta\omega)],$$

where $\hat{\psi}$ is a unit vector that describes the output field pattern, and θ is the angle between the two vectors at different frequencies. The first-order derivative of $C(\Delta\omega)$ with respect to $\Delta\omega$ is

$$\frac{dC(\Delta\omega)}{d\Delta\omega} = -\sin[\theta(\Delta\omega)]\frac{d\theta(\Delta\omega)}{d\Delta\omega}.$$

At $\Delta\omega = 0$, $\theta(\Delta\omega) = 0$ and $\sin[\theta(\Delta\omega)] = 0$. Thus the first-order derivative of the spectral correlation function

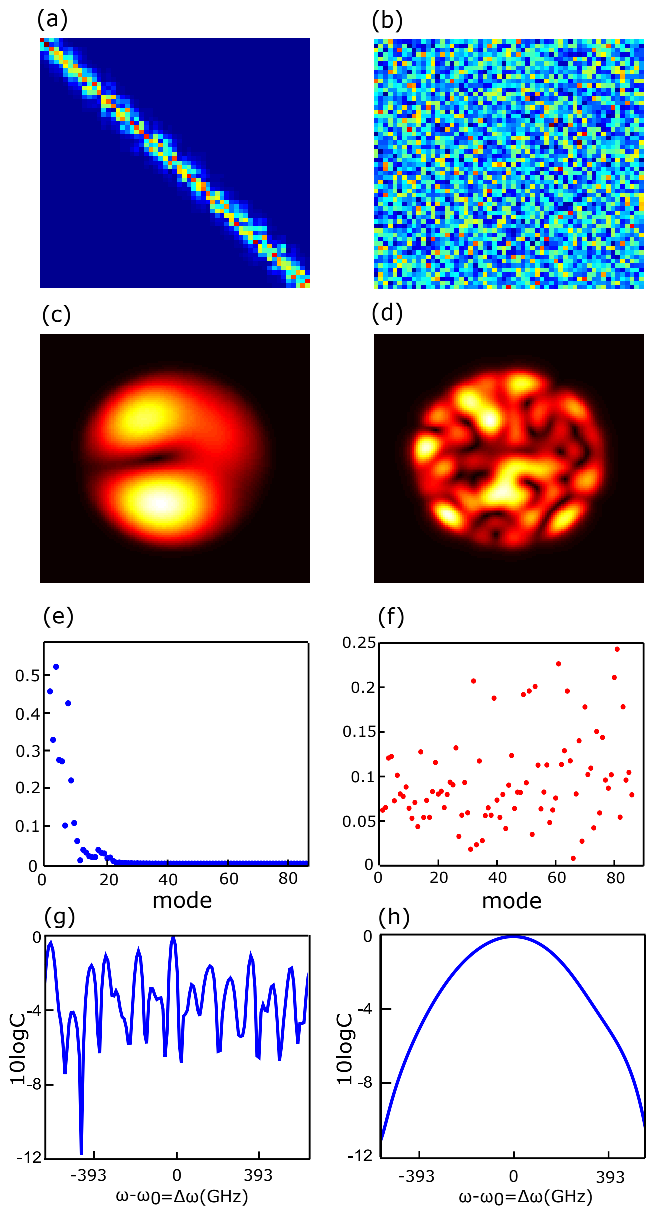

Figure S1. (color online) Comparison of multimode fiber with weak (a,c,e,g) and strong (b,e,f,h) mode coupling. The results are obtained by numerical calculation. (a,b) Transmission matrix in the mode basis. (c,d) Spatial distribution of output field for a PM. (e,f) Amplitude of the mode decomposition coefficients for the input fields of the PMs in (c,d). (g,h) Correlation of the output field pattern C with frequency detuning $\Delta\omega$ when light is injected to a few modes of the fiber.

vanishes for any input field. The second-order derivative of $C(\Delta\omega)$ is

$$\frac{d^2C(\Delta\omega)}{d\Delta\omega^2} = -\cos[\theta(\Delta\omega)]\left[\frac{d\theta(\Delta\omega)}{d\Delta\omega}\right]^2 - \sin[\theta(\Delta\omega)]\frac{d^2\theta(\Delta\omega)}{d\Delta\omega^2}$$

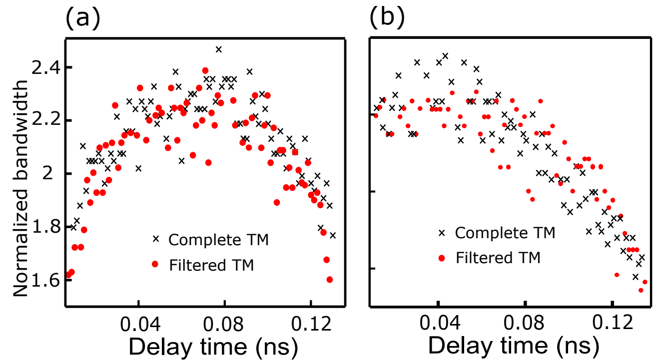

Figure S2. (color online) Spectral correlation width of the PMs for both polarizations (black cross) and those for one polarization (red circle) of input and output light in the multimode fiber with strong mode mixing. The width is normalized by the average correlation width for random input fields. The mode-dependent loss is absent in (a), and present in (b).

At $\Delta\omega = 0$, the second term on the right hand side vanishes, and $\cos[\theta = 0] = 1$ in the first term, giving $C''(\Delta\omega = 0) = -[\theta'(\Delta\omega = 0)]^2$.

For a PM, the output field pattern is invariant with frequency to the first order, i.e., $\theta'(\Delta\omega = 0) = 0$. Thus the second-order derivative of the spectral correlation function vanishes, $C''(\Delta\omega = 0) = 0$. For a random input field, $\theta'(\Delta\omega = 0)$ does not vanish, and hence $C''(\Delta\omega = 0)$ is nonzero. Consequently, the spectral decorrelation of PMs is much slower than that of a random input.

We note that C'' vanishes only at ω_0, not at nearby frequencies for a PM. Moreover, the third-order derivatives are non-zero even at ω_0. Thus the output field of a PM still de-correlates as a result of a frequency shift, leading to a finite bandwidth.

D. PARTICLELIKE STATE VS. PRINCIPAL MODES

Particle-like states [3, 4] and principal modes have the common feature that they are both eigenstates of the Wigner-Smith time-delay operator. The fact that the particle-like states additionally stay spatially focused throughout the scattering process distinguishes these two types of scattering states. To have particle-like states, however, the relevant length scales for wave spreading need to be longer than the system size, a requirement not fulfilled in elongated multimode fibers.

In our experiment, instead, light diffracts after propagating a certain distance in the fiber (characterized by the Rayleigh range), whereby all particle-like states are strongly depleted. Irrespective of this limitation, the eigenstates of the Wigner-Smith time-delay operator still exist in the fiber (even in the presence of strong scattering). In other words, despite the fact that the wave now travels not along a single particle trajectory, but along many different ones at once, an eigenstate of the Wigner-

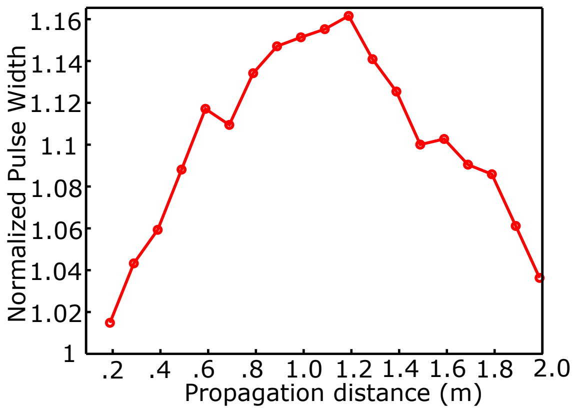

Figure S3. Calculated temporal width of an optical pulse at different location in a 2-meter-long fiber using the cancatenated fiber model. The input pulse is transform-limited and launched to the PM with the largest bandwidth. The pulse width is normalized to the input pulse. The pulse width first increases and then decreases as the pulse propagates in the fiber.

Smith matrix still has a well-defined delay time. This property, however, only holds at the end of the fiber and is not guaranteed anywhere between the two fiber facets. Correspondingly, the principal modes are not spatially focused throughout their propagation through the fiber.

We simulate the evolution of the temporal width of a transform limited pulse propagating through a multimode fiber. The input wavefront of the pulse is set to that of the principal mode with the largest bandwidth. The spectral bandwidth of the pulse, Δf, is 120 GHz, equal to the bandwidth of the principal mode. Figure S3 shows the temporal pulse width normalized by the width of the input pulse at different locations of the fiber. The pulse is broadened gradually in the fiber because of mode coupling. It becomes the broadest in the middle of the fiber. As it approaches the output end of the fiber, the pulse width decreases again. Figure S3 clearly shows that unlike particle-like states, principal modes do not remain focused in time throughout the fiber.

E. PM BANDWIDTH

We define the bandwidth of PMs as the width of the spectral correlation function at $C(\Delta\omega) = 0.9C(0)$. $2\pi/\Delta\omega$ determines the temporal width of an optical pulse that can transmit through the MMF without significant distortion of its shape at the end. The full width at half maximum of the spectral correlation function C gives a larger width than our definition, but the corresponding pulse suffers temporal distortions and no longer remains focused.

In Fig. 4(c), we show that PMs associated with short or long delay times have smaller bandwidths than those with medium delay times. In the concatenated fiber model, light propagates in each segment without mode coupling. Between adjacent segments, the modes are randomly coupled. We denote the length of the optical path associated with the m^{th} mode in the i^{th} segment as l_m^i. The total path-length for one trajectory through the fiber is $L = \sum_{i=1}^{N} l_m^i$, where N is the total number of segments. The probability of l_m^i taking the value of l_1^i, l_2^i, ..., or l_M^i is constant, where M is the highest order mode. According to the central limit theorem, when N is very large, L is a Gaussian distribution. Hence, there are more trajectories with intermediate total path-lengths in comparison to long or short paths. Since there are more trajectories available for a PM with an intermediate delay time, the multi-path interference effect is more efficient in narrowing the path-length distribution, making its bandwidth broader than that of a PM with a short or long delay time.

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