Effective PT-symmetric metasurfaces for subwavelength amplified sensing

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Abstract

We propose a novel design principle for ultrathin metasurfaces to realize optically amplified sensing with a performance that exceeds those of passive coherent perfect absorbers by several orders of magnitude. Our strategy is based on a generalized condition of lasing, coherent perfect absorption and their coexistence in metamaterials that feature an effective PT-symmetry. The devices we introduce here can be operated in configurations that involve both a one-sided or a two-sided wave incidence, where the latter case allows us to tune the degree of amplified absorption through the coherent phase between the two input beams. We also discuss how the conditions on the material parameters can be relaxed, away from the ideal case, such that a substantial amplification of the sensing performance can easily be reached in practical applications.

1. Introduction

Parity-time (PT) symmetric optics has recently emerged as a promising design principle for extending Hermitian to non-Hermitian optics and has given rise to a particularly rich variety of physical phenomena based on the appearance of exceptional points and phase transitions in the eigenvalues of the associated non-Hermitian Hamiltonians [1, 2]. The recent experimental realizations of PT-symmetric optical systems have attracted widespread interest, in particular due to their promising prospect to achieve tunable components with extreme sensitivity and very unconventional wave behavior [3–5]. These include power oscillations [6], PT-symmetric lasers [7–10], coherent laser-absorbers [11–13], unidirectional propagation [14–16] and optical solitons in PT periodic systems [17, 18], just to name a few of the numerous new concepts that have been put forward lately.

In parallel to the above developments, metamaterials, an array of artificial sub-wavelength electromagnetic resonators, have been shown to possess an almost unlimited palette of constitutive parameters for light manipulation [19–21]. It is thus a natural step to explore in which way metamaterials could serve as a generic platform to study PT-symmetry. In particular, by using metamaterials with both electric and magnetic responses, the possible wave phenomena that PT-symmetric metamaterials could give rise to, can be expected to be considerably richer than those in optical systems with an electric response only [22–28]. In this work, we show specifically that by employing generally bianisotropic metamaterials together with a redefinition of the parity operator, lasing, perfect absorption, and their co-existence [11] based on coherent external input can be naturally discussed in terms of an effective PT-symmetry. We will apply this insight to design ultrathin metasurfaces [29–33]—for optically amplified sensing applications, where the amplifying action comes from the lasing mode of a metasurface with appropriate gain and the sensing action comes from the material loss of the embedded sensor on the metasurface. Metasurface was firstly proposed as an ultra-thin layer of metamaterial with inhomogeneous material profile to control the transverse wavefront [29–33]. Recent works of metasurface have also extended the definition to any ultrathin slabs, consisting of one or a few layers of either homogenous or
inhomogeneous metamaterial atoms, which enjoy the same benefit of the earlier examples that fabrication becomes much easier and absorption is reduced due to the thin thickness [34–36].

The paper is organized in the following way. In section 2, we present the generalized condition of lasing, perfect absorption and their co-existence condition by the recently uncovered effective PT-symmetry in cross-matching the electric and the magnetic response [37]. Our considerations are generic in the sense that we only require a two-port system, e.g. one dimensional scalar wave propagation for stratified layers with decoupled transverse-electric or transverse magnetic polarization. The associated metamaterial can be generally bianisotropic with an arbitrary thickness. Based on the formulated conditions, we design a metamaterial device, which can be used to completely absorb light or can be used as a laser depending on the coherent inputs. We further demonstrate the corresponding realization using only one-sided incidence with either a perfect electric conductor (PEC) or a perfect magnetic conductor (PMC) as a back-plane. In section 3 we use this PT-symmetric device to propose a novel design of an optically amplified sensor with enhanced absorption comparing to a passive coherent perfect absorber (CPA) [38, 39]—a concept that we show to work also under non-ideal conditions. Both designs are based on ultrathin metasurfaces, so that the amplified sensor has a subwavelength size in the propagation direction. The proposed amplified sensor with effective PT-symmetry provides a unique way to achieve sensing devices with tunable and subwavelength characteristics. For example, the two-port amplified sensor can be used as a sensing device with additional tunability of the degree of amplification from the two coherent inputs. On the other hand, the sensing device, in its one-port configuration, can still achieve an amplification factor of over 400.

2. PT-symmetry in lasing, coherent perfect absorption and their co-existence

Our starting point is a generic two-port system composed of metamaterial slabs with an arbitrary total thickness \( L \) at normal incidence, as illustrated in figure 1(a). We assume that there is no cross-polarization coupling between transverse electric and transverse magnetic waves, corresponding to scalar wave propagation. Without loss of generality, we can consider in this case the polarization with electric field \( E_z \) and with magnetic field \( H_y \) while the slab extends normally to the \( z = -L/2 \) plane \((j = 1)\) and in the \( z = L/2 \) plane \((j = 2)\), respectively. (b)–(e) Schematic view of the \( P \) and \( T \) operation on CPA and lasing devices. The asterisk indicates complex conjugation.
complex amplitude of $H_{\mu}$ are in fact also $a_1$ and $b_2$ in Heaviside units, but an additional minus sign has to be added if the wave is propagating in the backward (negative $z$-) direction. Then, we can have the system response matrix $[37]$ defined as

$$C^{-1} = \frac{2}{i\phi_0} \frac{B - I}{S + I} B^{-1} \text{ with } B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

(1)

where $\phi_0 = k_0 L$ ($k_0$ is the vacuum wave number) and the scattering matrix $S$ is defined with convention $\{b_2, b_1\} = S \cdot \{a_1, a_2\}$. This matrix relates the difference of fields on the two sides to the averaged fields by

$$\begin{pmatrix} H_{\mu 2} - H_{\mu 1} \\ E_{\mu 2} - E_{\mu 1} \end{pmatrix} = \frac{i\phi_0}{2} C^{-1} \begin{pmatrix} E_{\mu 2} + E_{\mu 1} \\ H_{\mu 2} + H_{\mu 1} \end{pmatrix}.$$  (2)

We note that in the limit that the slab is much thinner than the wavelength, $C^{-1}$ can be regarded as the effective medium constitutive tensor of the system $[37, 40]$. For a general thickness, the definition of $C^{-1}$ in equation (1) is a generic system response matrix, similar to the impedance and ABCD matrices. In fact, the matrix $C^{-1}$ is Hermitian for a system without gain and loss and has eigenvalues with only non-negative imaginary parts for a totally passive system. It thus shares a certain equivalence to the Hamiltonian of a quantum mechanical system and can be used as a starting point to investigate PT-symmetry phenomena. For example, an eigenvalue degeneracy (exceptional point) of $C^{-1}$ with PT symmetry corresponds to unidirectional zero reflection $[37]$. Here, we employ this system response matrix to study the phenomenon of a PT laser-absorber $[1, 12]$—a device that can simultaneously emit light and perfectly absorb it. Suppose we label the constitutive matrix $C^{-1}$ as

$$C^{-1} = \begin{pmatrix} \epsilon + i\kappa & \mu \\ -i\kappa & \epsilon - i\kappa \end{pmatrix} = \begin{pmatrix} n + \Delta n & ik \\ -ik & n - \Delta n \end{pmatrix},$$

where $n = \frac{\epsilon + \mu}{2}$ and $\Delta n = \frac{\epsilon - \mu}{2}$. Then equation (1) can be used to relate a zero of $S$ matrix (coherent perfect absorption) to an eigenvalue $2i/\phi_0$ of the $C^{-1}$ matrix and a pole of $S$ matrix (lasing) to an eigenvalue of $-2i/\phi_0$ of the $C^{-1}$ matrix. There are actually two possible eigenvalues $n + K$ and $n - K$ (with $K = \sqrt{\Delta n^2 + k^2}$) for the $C^{-1}$ matrix. Hereafter, we use $`K`$ and $`-K`$ to label the two modes with eigenvalue $n + K$ and $n-K$ (for different mode profiles) respectively. A coherent perfect absorbing mode means either the eigenvalue of $K$ mode or $-K$ mode matches $2i/\phi_0$, while, similarly, a lasing mode means either of the two eigenvalues matches $-2i/\phi_0$. In a more compact form the condition of perfect absorption or lasing for our system can be summarized in the following form

$$\text{det}(C^{-1} + 2i/\phi_0 I) = (n + 2i/\phi_0)^2 - K^2 = 0,$$

(4)

where the upper (lower) sign stands for absorption (lasing). Such a form suggests that the laser and absorber modes are related by a time-reversal symmetry operation as realized here through a complex conjugation operation on equation (4). Suppose, e.g., that we start from a coherent perfect absorbed mode as shown in figure 1(b) with inputs $a_1$ ($a_2$) incident from the left (right) side. By taking a time-reversal operation (represented by the operator $T$) on the system response matrix and on the modes (i.e., the left and right input waves), the absorbed mode can be transformed to a lasing mode by taking the complex conjugate of the $C^{-1}$ matrix and of the wave amplitudes as well as by swapping the incident and outgoing waves, see figures 1(b) and (c).

Next, we consider the role of the parity operator in formulating the conditions of lasing and perfect coherent absorption. For the absorption to be perfect, only input waves to the slab are allowed to be present with no output waves traveling away from the slab. By putting equation (4) with the upper sign into the $S$-matrix (obtained from $C^{-1}$ by equation (1)), the perfectly absorbed modes can be obtained by finding the null-space of $S$, and is given by $a_1/a_2 = (\Delta n - i\kappa)/K$ for the $K$ mode and $-(\Delta n + i\kappa)/K$ for the $-K$ mode. The $K$ mode and the $-K$ mode are the two possible solutions of equation (4) with the upper sign. This result indicates that there can be different ways to realize a CPA mode. Based on this observation, we choose the parity operator such as to generate one of the perfectly absorbed modes from the other. Instead of using the conventional mirror operator $M$ in the propagating direction, we redefine the parity operator $P$ here with the help of an internal operation (based on the Poynting operator $\Pi^4$) that involves exchanging electric and magnetic fields, i.e.,

$$P \begin{pmatrix} H_{\mu} \\ E_{\mu} \end{pmatrix} = i\Pi M \begin{pmatrix} H_{\mu} \\ E_{\mu} \end{pmatrix} = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} H_{\mu} \\ E_{\mu} \end{pmatrix}$$  [37]. Now, when we carry out the $P$ operation on the absorber mode in figure 1(b), the $C^{-1}$ will be transformed by exchanging the permittivity and permeability, and the absorber mode will be transformed accordingly as $\left(\begin{array}{c} a_1 \\ a_2 \end{array}\right) \rightarrow \left(\begin{array}{c} \frac{a_1}{a_2} \\ \frac{a_2}{a_1} \end{array}\right)$ and vice versa (see illustrations in figures 1(b)–(d)). In this way an equivalent configuration of perfect absorption is generated with another set of material parameters and coherent inputs. An interesting observation in this context is that the requirement to

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4 Here, Poynting operator $\Pi$ appears in the expression for the time-averaged energy flow: $(H_{\mu} E_{\mu})^* \cdot \Pi \cdot (H_{\mu} E_{\mu})$. The combined operator $P$ is Hermitian ($-\Pi \Pi^\dagger = iM \Pi = -iM$ and unitary.)
have a perfectly absorbing system for both of the two possible excitation configurations, results in the condition that both of these configurations correspond to the same system, i.e., $\epsilon = \mu$. This is exactly the previously formulated impedance matching condition for a perfect absorber based on metamaterials [41].

Now, we pair up the $P$ and $T$ operations together. As the two operators commute with each other, the combined action can be displayed in tabular form as in figures 1(b)–(e). For example, a $P$ operation brings an absorber mode in figure 1(b) to another absorber mode in figure 1(d). A further $T$ operation on this absorber mode brings this to a laser mode in figure 1(e). As the $T$ operation transforms between laser and absorber modes and the $P$ operations transform from one set of coherent inputs to another set of linear-independent coherent inputs, one can realize a system exhibiting both lasing and perfect absorption, the laser-absorber (or CPA-laser), by requesting the system to be invariant after a combined PT operation, i.e. $\epsilon = \mu^*$, $\kappa = \kappa^*$. By requiring such a PT-symmetric system to satisfy equation (4), we obtain the general form of the effective constitutive matrix for a laser-absorber as (under the assumption of a real $\kappa$)

$$C^{-1} = \begin{pmatrix} \pm i\sqrt{\frac{\phi_0^2}{4} + \kappa^2} & i\kappa \\ -i\kappa & -\pm i\sqrt{\frac{\phi_0^2}{4} + \kappa^2} \end{pmatrix}$$

(5)

Here, the upper and lower sign indicate two different realizations for a laser-absorber. As an example, we now focus on a metamaterial realization of such a laser-absorber. For simplicity, we assume $\kappa = 0$ and have chosen the upper-sign realization in equation (5). This choice of parameters corresponds to a symmetric absorber mode as $
abla_1 = \begin{pmatrix} 1 \\ \end{pmatrix}$ and an anti-symmetric laser mode as $\nabla_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. In other words, a symmetric configuration of incident waves from the forward and backward directions will be completely absorbed while the system also has a lasing mode with anti-symmetric output radiations. Here, it is worth mentioning that the laser mode denotes an eigen-mode with lasing character. This does not mean the laser needs one certain configuration of incoming waves (for example anti-symmetric inputs in this paper) to lase, but actually means that the material will exhibit lasing phenomena with any input waves having a coupling with the laser mode and the symmetry of input waves can only affect the bandwidth of lasing mode. We will show later in figure 4(d) that such effect can lead to tunability in amplified sensor.

In terms of material parameters, such a PT symmetric laser-absorber can be realized by using a homogenous metamaterial slab with intrinsic impedance $Z_M = \sqrt{\mu_M/\epsilon_M}$ and refractive index $n_M = \sqrt{\mu_M/\epsilon_M}$. As shown in the left panel of figure 2(a), where thickness of the metamaterial is $L$, $\epsilon_M$ and $\mu_M$ are the slab permittivity and permeability. Then the $\epsilon$ and $\mu$ parameters of the resultant $C^{-1}$ matrix of the whole system can be found using equation (1) with the $S$ matrix (obtained from a transfer matrix method) as follows

$$\epsilon = \frac{2 \tan \left( n_M \phi_0 / 2 \right)}{Z_M \phi_0}$$

and

$$\mu = \frac{2 Z_M \tan \left( n_M \phi_0 / 2 \right)}{\phi_0}$$

(6)

By inserting equation (6) into the laser-absorber condition, we immediately obtain the material condition for the laser-absorber as $Z_M = -i$ and $\tan \left( n_M \phi_0 / 2 \right) = 1$. As an illustration, the reflectance with symmetric/anti-symmetric incident waves calculated using transfer matrix method is plotted in figures 2(b) and (c), respectively, for varying $\phi_0$ and $n_M$ (with fixed $Z_M = -i$). We, indeed, observe a pronounced absorbing (lasing) band with symmetric (anti-symmetric) incident waves at the laser-absorber condition (the dashed curve). More importantly, figure 2 shows that the laser-absorber can actually be realized for the case with $\phi_0 \ll 1$, indicating that it can be realized by a metasurface with subwavelength thickness. An interesting point to observe is that for symmetric (anti-symmetric) incidence, the tangential magnetic (electric) field at the central plane with $z = 0$ becomes zero. This means that for applications where only one-sided incidence is possible, the laser and absorber situation for the current system can be equivalently realized by inserting a PEC or PMC at $z = 0$ so that the system becomes a slab of only half of the original thickness together with a PEC or PMC backplane, see illustration in figure 2(a). It is important to note that we obtain the lasing condition by gradually increasing $n_M$, which is linked to the imaginary part of the permittivity and permeability. A positive (negative) imaginary part means material loss (gain). The required gain/loss parameter $n_M$ can be reduced by employing a system of larger thickness. On the other hand, a value in the order of $n_M \approx 10 \rightarrow 20$ in figure 2 can be realized by using metasurfaces with subwavelength thickness near resonance, for example, by employing active components in resonating microwave metamaterials [42]. We also note that the linear model adopted in this work is only able to predict the response of an active system from a zero $n_M$ up to the lasing threshold (lasing band in figure 2). We also note here that the gain/loss parameter at lasing threshold is proportional to the reciprocal of the thickness of the structure. Beyond such a lasing threshold, nonlinearity is expected to come in the picture to stabilize the lasing action.
3. Amplified sensing using metasurfaces

As we will now show, the concurrence of material gain and loss in the same system also opens up the possibility of optically amplified sensing. In such a case, the gain, from optically active components, is used to further amplify the power absorbed by a sensor. While a passive absorber with impedance matching can be utilized to achieve perfect absorption [41], a gain-assisted sensor can outperform even a passive perfect absorber in terms of the absorbed power. In a recent study it was demonstrated, e.g., that a sensor can absorb up to two times of the incident power in a PT-symmetric acoustic system, working at an exceptional point with unidirectional reflectionless propagation [43]. In our current context, we have already shown that a one-sided lasing configuration with an appropriate choice of the back-plane is equivalent to a PT-symmetric laser (lasing mode of the two-sided configuration with PT-symmetry). Hereafter, we will exploit such one-sided PT-symmetric device working near the laser condition for optically amplified sensing, where the gain part can amplify the input signal and the loss part is used as the sensor.

However, we note that the one-sided configuration (see figure 2(a)) contains gain/loss material in permittivity/permeability in the same material, which is very hard to realize in practice. We thus propose here a more realistic amplified sensor metasurface configuration (see figure 3(a)). This device contains only loss in layer-A (blue layer in figure 3(a)), representing the sensor, and only gain in layer-B (green layer in figure 3(a)), representing the active component for amplification, and a PEC back-plane. The A- and B- layers have material parameters $\epsilon_A = 2i\gamma, \mu_A = 0$, $\epsilon_B = 0$, $\mu_B = -i\gamma$, and the thickness of each layer is $L/4$. The equivalent ‘full-structure’ by mirroring the structure and removing the PEC back-plane, now with total thickness $L$, is shown in the right panel of figure 3(a). In this case, the effective constitutive tensor $C^{-1}$ can be obtained by inserting the $S$ matrix of the ‘full structure’ into equation (1): $C^{-1} = \begin{pmatrix} i\gamma & 0 \\ 0 & 4i\gamma/(\gamma^2 - 8) \end{pmatrix}$, which satisfies the PT laser-absorber condition when $\gamma = 2/\phi_0$ (see figure 3(b)). As illustrated in figure 3(d) the total absorption $A$ (the total dissipated power in layer A, normalized to the incident power), shows an amplified absorption band of the device near the one-sided lasing condition in figure 3(c). Here, we obtain an absorption efficiency, over $10^4$ times, much larger than that of a passive perfect absorber around the considered band.

Furthermore, we note that the discussed PT laser-absorber condition is overly restrictive for our purposes. As an example, consider here the geometric configuration in figure 4(a): layer-A (permittivity $\epsilon_A$ and permeability $\mu_A$), layer-B ($\epsilon_B$ and $\mu_B$) and a PEC back-plane. We can again retrieve the effective constitutive
matrix $C^{-1}$ of the equivalent ‘full-structure’ by mirroring the amplified sensor configuration, now with doubled thickness, right panel of figure 4(a). Figure 4(b) shows the retrieved effective $C_{11}^{-1}$ ($C_{22}^{-1}$) by choosing the material parameters as $\epsilon_A = -2 + 2i\gamma$, $\mu_A = 2$, $\epsilon_B = 2$, $\mu_B = -2 - i\gamma$. Additional real parts of material parameters can now be added (as a relaxed condition) to the configuration in figure 5, for more practical considerations in matching a realistic sensor and for more flexibility in constructing the amplifying sensor configuration. In the
thin slab limit (i.e., a metasurface), the additional real parts on the material parameters will not affect much on the position of the laser-absorber condition $\text{Im}(C_{11}^{-1}) \approx \frac{2}{\phi_0}$ and $\text{Im}(C_{22}^{-1}) \approx -\frac{2}{\phi_0}$, as shown as the solid red curves in figure 4(b), at $\gamma \approx 2/\phi_0$ (the vertical black dashed line). Generally, the non-zero real parts of the material parameters are expected to introduce additional contributions to the real parts of $C^{-1}$, e.g., to weaken the amplification performance. In our case, by taking a Taylor series expansion of $\phi_0$ on $C^{-1}$ at $\gamma = 2/\phi_0$, we found that the real parts of $C^{-1}$ are still very small, nearly zero, as shown in figure 4(b) by the solid black curves. The amplified sensor mode in figure 4(c) for one-side incidence on the ‘half-structure’ is thus still very prominent under such a relaxation of material parameters. In particular, the enhancement factor for the absorption at the amplifying sensing of the relaxed case is still as high as 400 times, as compared to a passive perfect absorber.

Before we get to the final conclusion, it is worth mentioning that for the ‘full-structure’ case with two-sided incidence, the lasing bandwidth can be tuned by the coherence between the two input waves as mentioned before. Therefore, by exploiting such a phenomenon, the absorption of the sensor can be tuned dynamically, shown in figure 4(d). When the inputs are symmetric and in-phase, the incident waves are completely absorbed by the system without reflection, acting like a passive perfect absorber. Now, when the relative phase is gradually changed from in-phase to out-of-phase (i.e., approximately to the laser mode), the absorption can be tuned from 1 to 400 (or over $10^4$ for the ideal case in figure 3). It is also worth to mention that the proposed PT laser-absorber may suffer practical limitations such as the spontaneous emission noise, which can decrease the signal sensitivity, and also saturation, which can decrease the actual achievable amplification ratio.

4. Conclusion

In conclusion, we have put forward the general conditions for lasing, coherent perfect absorption and for their co-existence using a framework of PT-symmetry based on cross-matching the electric and the magnetic response in a metamaterial. Based on such a PT-device, we have proposed novel designs for optically amplified sensors with substantially enhanced absorption compared to a CPA. We have also proposed an amplified sensor with relaxed material parameters, which can be constructed with present-day technology for ultra-thin metasurfaces.

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