Chaotic-to-regular crossover of shot noise in mesoscopic conductors

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Abstract. We study the shot noise by numerically simulating phase-coherent transport through a quantum dot. The chaotic-to-regular crossover regime of shot noise suppression is investigated explicitly by tuning the disorder potential and the openings of the dot. Employing the Modular Recursive Green's Function Method we obtain results for the Fano factor in regular systems which show a remarkable similarity to the results in chaotic systems. We argue that in the absence of chaotic scattering diffraction at the lead openings is the dominant source of shot noise. Estimates for the shot noise induced by this mechanism are presented, which agree with the numerical data.

Keywords: shot noise, quantum dots, ballistic transport
PACS: 73.23.-b, 05.45.Mt, 73.63.Kv, 72.70.+m

The current noise induced by the discreteness of the electron charge ("shot noise") has attracted attention now already for almost a century [1]. Recently this topic has resurfaced in the field of *mesoscopic physics* [2, 3], where ballistic quantum transport experiments [4, 5] and theoretical advances [6, 7, 8, 9, 10, 11, 12, 13] have mutually stimulated each other. In this context shot noise has been employed to explore the crossover from a deterministic (classical) particle picture of electron motion to a probabilistic (quantum) description, where electrons behave as matter waves. The quantum uncertainty inherent in the latter picture gives rise to noisy transport. In conductance through quantum dots the correlations between electrons in the Fermi sea lead to a suppression of shot noise S relative to the Poissonian value of uncorrelated electrons S_P [14] which is customarily expressed in terms of the Fano factor $F = S/S_P$.
Most investigations to date have focused on quantum dots whose classical dynamics is fully chaotic [6, 8, 9, 10, 11, 12, 14, 15, 16]. In this limit, random matrix theory (RMT) [16] predicts a universal value for the Fano factor, $F = 1/4$. The applicability of this RMT result requires, in addition to the underlying chaotic dynamics, dwell times in the open cavity τ_D which are sufficiently long compared to the Ehrenfest time τ_E. The latter estimates the time for the initially localized quantum wavepackets to spread all over the width d of the cavity (typically $d \approx \sqrt{A}$ with A area of the dot) due to the divergence of classical chaotic trajectories. It can be estimated as [17]

$$\tau_E = \Lambda^{-1} \ln(d/\lambda_F)\,, \qquad (1)$$

where Λ is the Lyapunov exponent ($\Lambda > 0$ for a chaotic cavity), and λ_F is the de Broglie wavelength associated with the wavenumber at the Fermi surface k_F. The limit $\tau_E/\tau_D \ll 1$ corresponds to the quantum (or RMT) regime and $\tau_E/\tau_D \gg 1$ corresponds to the classical limit for which $F = 0$ is expected. For ballistic cavities in the crossover

CP780, *Noise and Fluctuations: 18th International Conference on Noise and Fluctuations-ICNF 2005,* edited by T. González, J. Mateos, and D. Pardo

between these two regimes a simple conjecture for F was put forward [6],

$$F = 1/4 \exp\left(-\tau_E/\tau_D\right) . \tag{2}$$

For cavities with a short-ranged disorder potential, an alternative crossover behavior,

$$F = 1/4 \left(1+\tau_Q/\tau_D\right)^{-1} , \tag{3}$$

was proposed [4, 11], where τ_Q is a characteristic scattering time within which the wavepacket is scattered into random direction. The quantities τ_Q and τ_E are closely related to another as both denote the characteristic time scale for spreading of the wavepacket by chaotic scattering either at the boundary (τ_E) or the interior (τ_Q) of the cavity. Moreover, for short ranged disorder with a correlation length $l_C < \lambda_F$, τ_Q incorporates, just as τ_E, quantum effects and depends on an effective $\hbar_{\rm eff}$ of the system. The crossover from the chaotic to the regular regime is therefore predicted to be controlled by a single ratio τ_E/τ_D or τ_Q/τ_D which will be a function of the size of quantum effects ($\hbar_{\rm eff}$) and the mean rate of irregular (chaotic) scattering, $\langle\Lambda\rangle$. The chaotic-to-regular crossover corresponds to the limit $\langle\Lambda\rangle \to 0$, while the quantum-to-classical limit involves $\hbar_{\rm eff} \to 0$.

One open question not yet well understood is the behavior of shot noise for motion in a regular rather than chaotic cavity, i.e. in the limit $\Lambda \to 0$. For a mixed system lower values of F have been observed [7] suggesting that for regular systems F may vanish. Taken at face value, Eq. (2) yields $F = 0$ (complete suppression of shot noise) for the case of $\tau_E \to \infty$ or $\Lambda \to 0$ at fixed value of $\hbar_{\rm eff}$. To investigate this question, we analyze a model system that allows to study the crossover regime from chaotic to regular dynamics, i.e. $\Lambda \to 0$, explicitly. Our scattering system consists of a rectangular cavity to which two leads of width d are attached via tunable shutters with an opening width w (see Fig. 1a). Varying the lead openings and the disorder potential allows to tune the dwell time τ_D and the mean rate of chaotic spreading of the wavepacket, $\langle\Lambda\rangle$, independently. The cavity region of width d and length $2d$ contains a disorder potential V characterized by its mean value $\langle V\rangle = 0$, and the correlation function $\langle V(x)V(x+a)\rangle = \langle V^2\rangle \exp(-a/l_C)$. The correlation length l_C is typically a small fraction of the Fermi wavelength $l_C/\lambda_F \approx 0.12$ and the potential strength $V_0 = \sqrt{\langle V^2\rangle}$ is weak, $V_0/E_F \le 0.1$. In the limit of vanishing disorder $(V_0 \to 0)$ the motion inside the cavity becomes completely regular.

Our quantum calculation proceeds within the framework of the modular recursive Green's function method (MRGM) [18] which allows to treat two-dimensional quantum dots with relatively small λ_F (or small $\hbar_{\rm eff}$). Details are given in Ref. [13]. We evaluate the transmission amplitudes t_{mn} for an electron injected from the left by projecting the Green's function at the Fermi energy $G(E_F)$ onto all modes $m,n \in [1,\ldots,N]$ in the in- and outgoing lead, respectively. The Fano factor F is then calculated from the N-dimensional transmission matrices t [3],

$$F = \frac{\langle \mathrm{Tr}\, \mathrm{t}^\dagger \mathrm{t}(\mathbb{1}-\mathrm{t}^\dagger \mathrm{t})\rangle}{\langle \mathrm{Tr}\, \mathrm{t}^\dagger \mathrm{t}\rangle} = \frac{\langle \sum_{n=1}^{N} T_n(1-T_n)\rangle}{\langle \sum_{n=1}^{N} T_n\rangle} , \tag{4}$$

with T_n being the eigenvalues of $t^\dagger t$. The brackets $\langle\ldots\rangle$ indicate that we average over 150 equidistant points in the wavenumber-range $k_F \in [40.1, 40.85] \times \pi/d$, where 40 transverse lead modes are open. Figure 1c displays the Fano factor as a function of the inverse dwell time τ_D^{-1}. For $\tau_D^{-1} \to 0$ (i.e. large dwell times) F approaches the universal value 1/4 irrespective of the strength of the disorder potential V_0, while for shorter dwell times F falls off gradually (Fig. 1c). The steepness of this decrease is clearly dependent on V_0 and thus on the mean scattering rate $\langle \Lambda \rangle$. Most striking is the feature that for $V_0 \to 0$ but long dwell times the shot noise reaches the RMT value even though the dynamics is now entirely regular (see Fig. 1c). This observation suggests that the conjectures [Eq. (2) or Eq. (3)] require modifications to properly account for the shot noise in the regular limit. We argue that the key point is the wavepacket diffraction at the cavity openings which has to be incorporated in the theoretical description of shot noise [19, 20, 21]. Note that this feature is inherent in quantum transport and independent of the underlying regular or chaotic dynamics [18]. Scattering due to *chaotic* dynamics, which lies at the core of RMT, certainly leads to wavepacket spreading but does not constitute the only or, in general, dominant source.

To quantify the amount of diffraction in the cavity we perform a quasi-classical Monte-Carlo transport simulation in which we follow an ensemble of classical trajectories subject to Fraunhofer scattering at the shutter openings [19] and a random Poissonian scattering process in the disorder potential region [22]. For the latter we calculate the transport mean free path ($\tau_S \cdot v_F$) and the differential scattering probability ($P(\theta) \sim d\sigma/d\theta$) in first Born approximation, thus taking into account quantum diffractive scattering (for $l_C \cdot k_F < 1$) along the lines of Refs. [4, 11]. We find that the differential cross section is strongly peaked at small forward scattering angles (see Fig. 1b). The modified Ehrenfest time $\widetilde{\tau}_E$ which includes these diffractive corrections is drastically reduced as compared

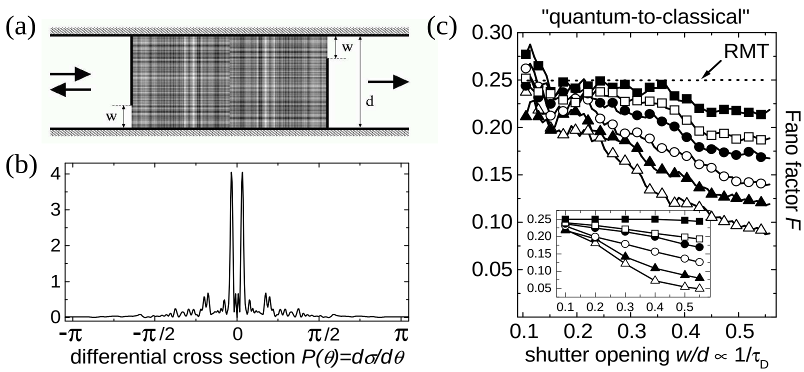

FIGURE 1. (a) Rectangular quantum billiard with tunable shutters and disorder potential (gray shaded area). Tuning the opening of the shutters w, the crossover from quantum-to-classical scattering can be investigated. (b) Normalized differential scattering probability $P(\theta) \sim d\sigma/d\theta$ as calculated in first Born approximation for the employed disorder potential. (c) Fano factor F in the quantum-to-classical crossover regime (numerical data from the full quantum simulation for the geometry depicted in (a)). Curves shown correspond to different strengths of the disorder potential (measured with respect to the Fermi energy E_F): $V_0/E_F = 0.1\,(\blacksquare),\ 0.07\,(\square),\ 0.05\,(\bullet),\ 0.03\,(\circ),\ 0.015\,(\blacktriangle),\ 0\,(\triangle)$. A decrease from the "quantum value" $F = 1/4$ for large τ_D towards the "classical value" $F = 0$ for short τ_D is clearly visible. The inset depicts the theoretical prediction based on a quasiclassical simulation. Note the good agreement with the numerical data from the full quantum calculation.

to the conjecture in Eq. (1). For an improved estimate of the Fano factor F we additionally take into account the exact dwell time distribution $P(t)$, resulting from the quasi-classical simulation for the particular system we study. Following [9] these ingredients determine the Fano factor F as follows:

$$F = 1/4\left[1 - \int_0^{\tilde{\tau}_E} P(t)\,dt\right] = 1/4\int_{\tilde{\tau}_E}^{\infty} P(t)\,dt\,. \tag{5}$$

Note that this expression is applicable to chaotic as well as regular systems and valid irrespective of whether the origin of spreading is ballistic scattering at the boundary or diffractive scattering inside the cavity. The estimate according to Eq. (5) (see inset of Fig. 1c) is in very good agreement with the results from the quantum calculations.

To summarize, we have numerically determined the behavior of the Fano factor F in a realistic scattering system with a tunable disorder potential and tunable shutters. We find that diffraction at the lead openings is sufficient to establish the RMT prediction for shot noise suppression ($F = 1/4$), irrespective of regular or chaotic dynamics. The chaotic-to-regular crossover in F can be estimated by a generalization of a previously proposed dependence [9] on the Ehrenfest time $\tilde{\tau}_E$ [Eq. (5)], provided that the definition of the Ehrenfest time is properly modified to include diffraction.

We thank C. W. J. Beenakker, J. Cserti, V. A. Gopar, F. Libisch, I. Rotter, H. Schomerus, M. Seliger, E. V. Sukhorukov and L. Wirtz for helpful discussions. Support by the Austrian Science Foundation (Grant No. FWF-SFB016 and No. FWF-P17359) is gratefully acknowledged.

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