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Shot noise in transport through quantum dots: Clean versus disordered samples

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Abstract We investigate the role of disorder and diffractive scattering in the shot noise power of quantum transport through a two-dimensional quantum dot. By tuning the strength of the disorder potential and the openings of the dot, we numerically explore the influence of quantum scattering mechanisms on the current shot noise. For small cavity openings we find the shot noise for disordered samples to be of almost equal magnitude as for clean samples where transport is ballistic. We explain this finding by diffractive scattering at the cavity openings which act as strong noise sources. Estimates for the shot noise induced by both the disorder potential and the diffractive openings are presented that agree with the numerical data.

Keywords Shot noise · Quantum dots · Disorder · Ballistic transport

1 Introduction

Now almost a century ago Walter Schottky investigated the current noise in vacuum tubes and explained this phenomenon ("shot noise") by the granularity of the electron charge [1]. Recently this topic has resurfaced in the field of *mesoscopic physics* [2, 3], as it was shown that the phase-coherent current through low temperature conductors shows very distinct shot noise characteristics. Ballistic quantum transport experiments [4, 5] as well as theoretical investigations [6–13] have demonstrated that the time-dependent current shot noise contains very useful information about the system properties of a conductor. This information has, e.g., been used to explore the crossover from a deterministic (classical) particle picture of electron motion to a probabilistic (quantum) description, where electrons behave as matter waves [4, 7]. Contrary to the classical vacuum tube where the noise is due to the random emission of charges, it is the quantum uncertainty inherent in phase-coherent transport which gives rise to current shot noise in mesoscopic systems. Due to the correlations between electrons in the Fermi sea the mesoscopic shot noise S is suppressed relative to the Poissonian value of uncorrelated electrons S_P [6]. This suppression is customarily expressed in terms of the Fano factor $F = S/S_P$.

Most investigations to date have focused on mesoscopic quantum dots whose dynamics is fully chaotic or fully stochastic. In this limit random matrix theory (RMT) [14] predicts a universal value for the Fano factor, $F = 1/4$. The applicability of this RMT result requires, however, dwell times in the open cavity, τ_D, which are sufficiently long compared to the characteristic quantum scattering time τ_Q in the cavity potential. The latter estimates the time for the initially localized quantum wavepackets to be scattered into random direction and thus to be spread all over the width d of the cavity (typically $d \approx \sqrt{A}$ with A area of the dot). The limit $\tau_Q/\tau_D \ll 1$ corresponds to the quantum (or RMT) regime and $\tau_Q/\tau_D \gg 1$ corresponds to the classical limit for which $F = 0$ is expected. For cavities where τ_Q is determined by stochastic quantum scattering, a simple conjecture for F was put forward [4, 11],

$$F \approx 1/4\,(1 + \tau_Q/\tau_D)^{-1}\,. \tag{1}$$

We note at this point that a similar prediction was presented for the case that the spreading of the wavepacket in the cavity is due to classical chaotic dynamics [7]. In the case we study here, the noise source is however given by a short-range

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disorder potential and Eq. (1) should be applicable. The term "short-range" refers to the correlation length of the disorder, l_C, which is smaller than the Fermi wavelength λ_F. The scattering of wavepackets at this potential is thus determined by stochastic quantum diffraction rather than by classical scattering mechanisms. The strength and shape of the disorder potential will control the amount of diffractive scattering that enters τ_Q. The shot noise and its crossover from the disordered (quantum) regime to the clean (classical) regime is then determined by the ratio τ_Q/τ_D in Eq. (1).

An obvious test of the above prediction would be to simulate phase-coherent scattering processes on a computer. However, conventional numerical techniques suffer from a slow convergence rate, especially for the case where many open modes participate in the transport process. To circumvent this difficulty an "open" dynamical kicked rotator model was recently successfully used to mimic chaotic and stochastic scattering in a 1D-system [9, 10, 15]. Experimental tests have, so far, been limited to the regime of $\tau_Q/\tau_D < 1$ [4].

2 Numerical simulation

In the present article we explore the disordered-to-clean crossover of shot noise by numerically simulating the full two-dimensional quantum transport problem for the case of a disordered cavity. In particular, we investigate the predicted suppression of shot noise for decreasing disorder strength. Taken at face value, Eq. (1) yields $F = 0$ (complete suppression of shot noise) for the case of negligible scattering rates, i.e. for $\tau_Q \to \infty$. To investigate this question, we analyze a model system that allows to study the crossover regime from disordered to regular dynamics explicitly.

Our scattering system consists of a rectangular cavity to which two leads of width d are attached via tunable shutters with an opening width w (see Fig. 1(a)). Varying the lead openings and the disorder potential allows to tune the dwell time τ_D and the mean rate of spreading of the wavepacket independently.

The cavity region of width d and length $2d$ contains a disorder potential V characterized by its mean value $\langle V \rangle = 0$, and the correlation function $\langle V(x)V(x+a)\rangle = \langle V^2\rangle \exp(-a/l_C)$. The correlation length l_C is typically a small fraction of the Fermi wavelength $l_C/\lambda_F \approx 0.12$ and the potential strength $V_0 = \sqrt{\langle V^2\rangle}$ is weak, $V_0/E_F \le 0.1$. In the limit of vanishing disorder ($V_0 \to 0$) the motion inside the cavity becomes completely regular.

Our quantum calculation proceeds within the framework of the modular recursive Green's function method (MRGM) [16] which allows to treat two-dimensional quantum dots with relatively small λ_F (details are given in Ref. [12]). We evaluate the transmission amplitudes t_{mn} for an electron injected from the left by projecting the Green's function at the Fermi energy $G(E_F)$ onto all modes $m, n \in [1, \ldots, N]$

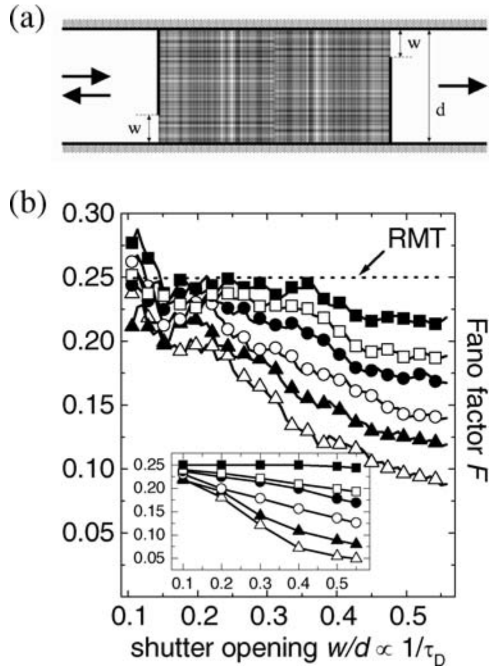

Fig. 1 (a) Rectangular quantum billiard with tunable shutters and disorder potential (gray shaded area). Tuning the opening of the shutters w, the dwell time τ_D in the cavity can be varied. (b) Fano factor F in the quantum-to-classical crossover regime (numerical data from the full quantum simulation for the geometry depicted in (a)). Curves shown correspond to different strengths of the disorder potential (measured with respect to the Fermi energy E_F): $V_0/E_F = 0.1\,(\blacksquare)$, $0.07\,(\square)$, $0.05\,(\bullet)$, $0.03\,(\circ)$, $0.015\,(\blacktriangle)$, $0\,(\triangle)$. A decrease from the "quantum value" $F = 1/4$ for large τ_D towards the "classical value" $F = 0$ for short τ_D is clearly visible. The inset depicts the theoretical prediction based on a quasiclassical simulation. Note the good agreement with the numerical data from the full quantum calculation

in the in- and outgoing lead, respectively. The Fano factor F is then calculated from the N-dimensional transmission matrices t [3],

$$F = \frac{\langle \mathrm{Tr}\ t^\dagger t(1 - t^\dagger t)\rangle}{\langle \mathrm{Tr}\ t^\dagger t\rangle} = \frac{\left\langle \sum_{n=1}^{N} T_n(1 - T_n)\right\rangle}{\left\langle \sum_{n=1}^{N} T_n\right\rangle}, \tag{2}$$

with T_n being the eigenvalues of $t^\dagger t$. The brackets $\langle \ldots \rangle$ indicate that we average over 150 equidistant points in the wavenumber-range $k_F \in [40.1, 40.85] \times \pi/d$, where 40 transverse lead modes are open.

3 Results and discussion

The numerical results for the Fano factor F are displayed in Fig. 1(b) as a function of the inverse dwell time τ_D^{-1}. For $\tau_D^{-1} \to 0$ (i.e. large dwell times) F approaches the universal value 1/4 irrespective of the strength of the disorder potential V_0, while for shorter dwell times F falls off gradually. The steepness of this decrease is clearly dependent on V_0 and thus on the strength of disorder scattering. Most striking is the feature that for $V_0 \to 0$ but long dwell times the shot noise reaches the RMT value even though the dynamics is now entirely regular (see Fig. 1(b)). This observation suggests that the quantum scattering time τ_Q contains additional contributions other than from disorder scattering alone. We argue that the key point is the wavepacket diffraction at the cavity openings which has to be incorporated in the theoretical description of shot noise [17–19]. Note that this feature is inherent in quantum transport and independent of the disorder potential in the cavity [16]. Disorder scattering certainly leads to wavepacket spreading but does not constitute the only or, in general, dominant source. We will show below how to modify our estimate for the quantum scattering time τ_Q to properly include diffraction of the cavity openings.

To quantify the amount of diffraction in the cavity we perform a quasi-classical Monte-Carlo transport simulation in which we follow an ensemble of classical trajectories subject to Fraunhofer scattering at the shutter openings [17] and a Poissonian random scattering process in the disorder potential region [20]. Diffraction at the cavity openings has been studied in detail [17, 18] and can be described by a standard Fraunhofer diffraction analysis for electrons which enter or leave the cavity [17]. To assess the Poissonian scattering off the disorder we calculate the transport mean free path ($\tau_S \cdot v_F$) and the differential scattering probability ($P(\theta) \sim d\sigma/d\theta$) in first Born approximation, thus taking into account quantum diffractive scattering (for $l_C \cdot k_F < 1$) along the lines of Refs. [4, 11]. Our simulation contains wave spreading resulting from both the Fraunhofer injection of classical trajectories at the shutter opening and from multiple disorder scattering inside the cavity. The modified quantum scattering time $\tilde{\tau}_Q$ which includes the Fraunhofer corrections is drastically reduced as compared to τ_Q which contains contributions from disorder scattering alone. For an improved estimate of the Fano factor F we additionally take into account the exact dwell time distribution $P(t)$, resulting from the quasi-classical simulation for the particular system we study. Following [8] these ingredients determine the Fano factor F as follows:

$$F = 1/4\left[1 - \int_0^{\tilde{\tau}_Q} P(t)\,dt\right] = 1/4 \int_{\tilde{\tau}_Q}^{\infty} P(t)\,dt\,. \qquad (3)$$

The above estimate (see inset of Fig. 1b) is in very good agreement with the results from the quantum calculations.

4 Summary

To summarize, we have numerically determined the behavior of the Fano factor F in a realistic scattering system with a tunable disorder potential and tunable shutters. We find that diffraction at the lead openings is sufficient to establish the RMT prediction for shot noise suppression ($F = 1/4$), irrespective of disorder in the cavity. The disordered-to-clean crossover in F can be estimated by a generalization of a previously proposed dependence [8] on the quantum scattering time τ_Q [Eq. (3)], provided that the definition of τ_Q is properly modified to include diffraction at the cavity openings.

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