Lasing in chaotic and random scattering media

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Abstract: Application of the ab-initio self-consistent (AISC) laser theory to multi-mode chaotic and random lasing media is presented. © 2008 Optical Society of America

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The semiclassical laser theory embodied in its most basic form in Maxwell-Bloch (MB) equations describes the oscillations of a majority of laser systems known today. It does so by coupling the electromagnetic field, described by Maxwell's equations to the non-linear polarization of the gain medium, described by quantum equations of motion. This theory describes all of the characteristic non-linear phenomena observed in lasers such as mode competition and selection, frequency pulling/pushing, bistability, synchronization, frequency and phase locking.

The laser is intrinsically an open system, and the electric field in a laser is generated by the stimulated emission of light from a collection of quantum emitters which are incoherently driven. The stationary electric field in the laser cavity does not conserve energy; the flow of light energy (magnitude of the Poynting vector) is increasing along the cavity in the direction of the output(s). Therefore the electric field in the cavity has a non-hermitian character and can be shown to be determined by a non-hermitian boundary condition; outside the cavity there is no gain medium and energy flux is conserved. This non-hermitian character of the actual lasing modes has been widely neglected in semiclassical laser theory which has until very recently only dealt with hermitian closed cavities in the multi-mode regime (e.g. [1]). Heuristic arguments were used to discuss the output power of the laser and its emission pattern.

Modern nanofabrication capabilities along with potential applications have led to a wide variety of new microlasers with complex cavity structures. We focus here on deformed dielectric cavity lasers and random nano-composite lasers. For such lasers the emission pattern and output power is not easily guessed from internal mode properties of the closed resonator. Thus we sought a more rigorous and predictive framework for analyzing arbitrarily complex and open multi-mode lasers. Our approach [2-4], the Ab Initio Self-Consistent (AISC) laser theory, is able to treat lasers with any degree of openness, and with arbitrarily strong multi-mode non-linear interactions. Here, we will review this new approach and describe its application to chaotic and disordered scattering media.

In Refs. [2-4] we have shown that starting from Maxwell-Bloch equations and a general multi-periodic form for the electric field $E(\boldsymbol{x},t) = \sum_\mu \Psi_\mu(\boldsymbol{x}) e^{-ik_\mu t}$, the stationary non-linear lasing modes can be obtained by the fixed points of the flow equations for the complex vector of components of the Constant Flux (CF) states, $\boldsymbol{a}^\mu$:

$$a_m^\mu = \frac{iD_0\gamma_\perp}{(\gamma_\perp - i(k_\mu - k_a))} \frac{(k_\mu^2/k_a^2)}{(k_\mu^2 - k_m^{\mu\,2})} \int d\boldsymbol{x}' \frac{\bar{\varphi}_m^{\mu *}(\boldsymbol{x}') \sum_p a_p^\mu \varphi_p^\mu(\boldsymbol{x}')}{n^2(\boldsymbol{x}')(1 + \sum_\nu \Gamma(k_\nu)|\Psi_\nu(\boldsymbol{x})|^2)} \,. \tag{1}$$

where $\Psi_\mu(\boldsymbol{x}) = \sum a_m^\mu \varphi_m^\mu(\boldsymbol{x})$, $\varphi_m^\mu(\boldsymbol{x}) = \varphi_m(\boldsymbol{x}, k_\mu)$ are the CF modes and $k_m^\mu = k_m(k_\mu)$ are the complex CF wavevectors parametrically dependent on the laser oscillation frequencies k_μ ($c = 1$). Here, $\bar{\varphi}_m^\mu$ are the adjoint functions to the φ_m^μ, $\Gamma(k)$ is the gain profile, k_a is the atomic transition frequency, $\gamma_\perp$ is the homogeneous broadening of the gain medium and D_0 is the pump strength. These equations are able to take into account an arbitrary degree of openness of a "cavity" described by a spatially varying index of refraction $n(\boldsymbol{x})$. Spatial hole-burning interactions in the multi-mode regime are included to infinite order.

Past theoretical and experimental work has shown that resonator shape is highly significant in controlling the emission characteristics (spectral as well as spatial) of deformed dielectric cavity lasers. Due to phase coherence, interference effects become important and strongly affect the device characteristics. For instance, for a particular class of resonators called ARCs, a smooth deformation of the device interface from a symmetric shape leads to a classical transition from integrability to chaos in its ray motion. Contrary to intuitive expectations however, the interplay of ray chaos and interference, which is a defining aspect of a general class

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of systems known as wave-chaotic systems, can lead to better device performance. A particular example are the results of Ref. [5] on deformed quantum cascade lasers, demonstrating that mere boundary deformation of a disk-like shape can lead to almost three orders of magnitude increase in output power. A theoretical understanding of the tremendous increase in efficiency of these devices remained elusive so far. Here we will argue that spatially non-uniform distribution of the pump power may play a crucial role in the dramatic increase in power efficiency of these devices.

Another major puzzle to which AISC theory provides a deeper understanding is the nature of lasing modes of a disordered scattering medium in the diffusive (delocalized) regime when the linear scattering problem has no distinct resonances (cavity finesse f is parametrically smaller than unity), yet the lasing modes have narrow spectral lines and Poisson photon statistics well above threshold [6]. We find [7] that the lasing modes away from the threshold are "composite" modes, which thus do not correspond to any single solution of the linear wave equation and show that lasing in the diffusive regime can be coherent but its properties are dominated by strong non-linear interaction between modes. We demonstrate that the interactions between lasing modes can be so strong that the frequency spectrum is "cleaned up" of modes with small frequency spacing (via a variant of spontaneous frequency locking or synchronization) so that the frequencies in a typical multimode DRL are well separated and dominated by collective effects. Furthermore, we are able to explain recent experiments on DRLs [8] which showed that the lasing intensities are extremely sensitive to pump fluctuations while the frequencies remain surprisingly immune. To access this regime we define and numerically solve the following model: A circular disk-shaped gain-medium of radius R, which contains randomly distributed dielectric particles of radius r_s, where $r_s \ll R, 2\pi/k_a$ (this is similar to the ZnO nanoparticle clusters of Ref. [6] except that we take the clusters to terminate on a circular boundary and assume gain everywhere in the cluster for convenience).

1. References

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