Semiclassical theory for transport through clean quantum dots: from qualitative reasoning to quantitative agreement

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Abstract

Within Feynman's formulation of quantum mechanics transport properties of quantum billiards can be understood as the result of path interference. We use two-dimensional Fourier-transforms ("length-area spectra") of the quantum mechanical transport amplitudes to gain information on contributing paths and their weights. We present a semiclassical theory that can account for quantum mechanical transport properties (weak localization, conductance fluctuations) on a quantitative level provided all relevant classical and non-classical contributions to the length-area spectra are represented.

Motivation

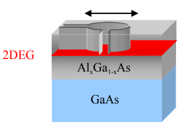
- ***Semiclassical theories*** are intuitive – transport as interference of paths (classically regular/ chaotic dynamics enters).

BUT

- standard theories ***do not give quantitative results*** [1]
- no prediction for weak localization in regular billiards

Transport theory

- 2D electron gas in a clean semiconductor hetero-structure

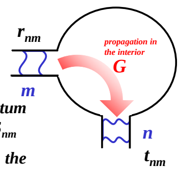


Phase coherent transport

- **Landauer formula**

$$g(k,B)=\frac{2e^2}{h}T(k,B)=\frac{2e^2}{h}\sum_{m,n}^{N}|t_{nm}(k,B)|^2 \quad (1)$$

g(k,B)...... conductance
T(k,B)...... total transmission
k,B......... wave number, magnetic field
N........... number of modes in the leads
t_{nm}......... transmission amplitude from mode m to n

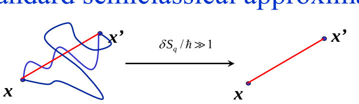


Transport determined by quantum mechanical S-matrix elements S_{nm}

Projection of modes m, n onto the Green's function G

- Numerical solution within tight-binding discretization – accurate but does not provide intuitive insight
- Alternative semiclassical approximation since $S_q/\hbar \gg 1$

Standard semiclassical approximation



- **Semiclassical Green's function**

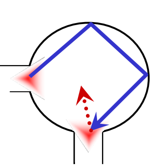
$$G_{SCA}=\frac{2\pi}{(2\pi i)^{3/2}}\sum_{\text{classical paths: }q}\sqrt{|D_q|}\,e^{i\left[\frac{S_q}{\hbar}-\frac{\pi}{2}\mu_q\right]} \quad (2)$$

$$S_q = kL_q + Ba_q/c \quad (3)$$

S_q, D_q, μ_q........................ classical action, deflection factor, Maslov index

L_q, A_q.................. length, directed enclosed area (can have positive and negative values) of path q

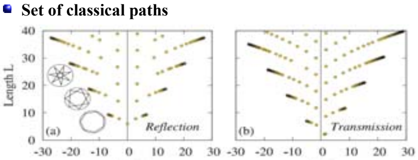
- **Coupling of leads**
  - **Classical**: no change in direction of propagation when entering/exiting (λ→0)
  - **Diffractive**: probability distribution of angles (projection integrals solved numerically or within theories of diffraction)



Standard semiclassics : after entering and before exiting the cavity the particle follows classical paths

Backscattered paths not included!!!

- **Set of classical paths**

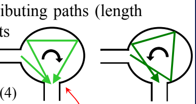


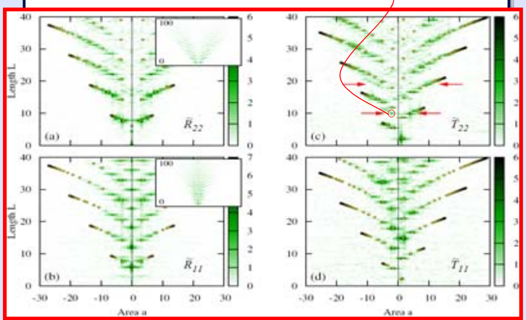
Time-reversal symmetry weak localization peak ***Off-set between branches no weak localization dip***

Quantum mechanical spectra

- **Fourier transform of the exact quantum S-matrix elements**
  - Contains the information on contributing paths (length and enclosed area) and their weights

$$\tilde{S}_{nm}(L,a)=\iint dk\,dB\,S_{nm}(k,B)e^{-i(kL+Ba/c)} \quad (4)$$



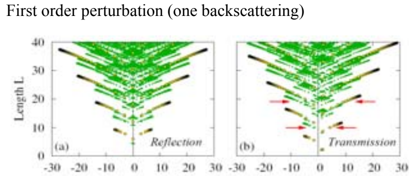


- **Non-classical paths are important**

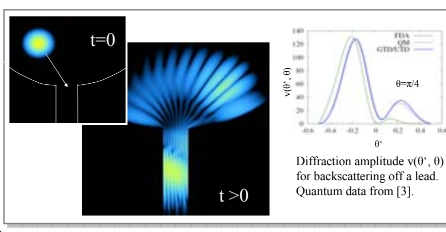
Pseudo-path semiclassical approximation

$$G_{PSCA}=G'_{SCA}+\sum_i (G'_{SCA}V)^i G'_{SCA} \quad (5)$$

- First order perturbation (one backscattering)



- **Improved diffraction theory (beyond Fraunhofer diffraction approximtion)**
  - Geometric theory of diffraction [2] (GTD) extended by uniform theory of diffraction [2] (UTD)

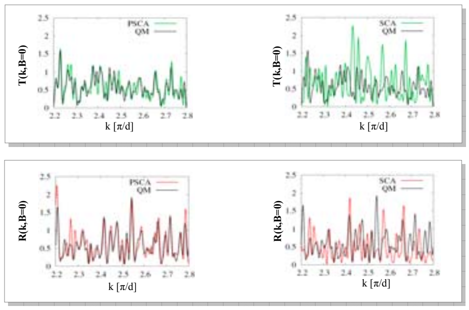


Diffraction amplitude v(θ', θ) for backscattering off a lead. Quantum data from [3].

Results

Conductance & Resistance

- **Pseudo-path semiclassical approximation (PSCA)** **Standard semiclassical approximation (SCA)**

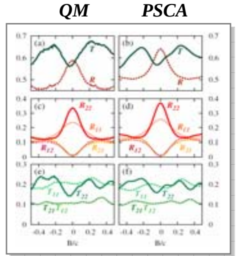


Fluctuations reproduced within PSCA with high accuracy.

Weak localization

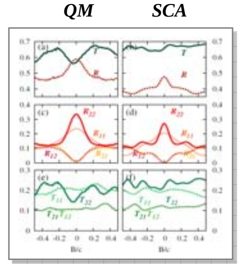
- **k-averaged transmission and reflection [4]**

$$\left\langle |t_{nm}(k,B)|^2\right\rangle_{\Delta k}=T_{nm} \qquad \left\langle |r_{nm}(k,B)|^2\right\rangle_{\Delta k}=R_{nm}$$



Dip in T reproduced

No Dip in T



Weak localization dip and all mode-to-mode probabilities reproduced on a quantitative level.

Conclusions

- Quantum mechanical path spectra give information on contributing paths
- This information can be implemented into a semiclassical theory
- Previously not considered quantum effects (diffraction) prove to be essential for weak localization

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