# Quantitative description of coherent transport through surface-disordered wires


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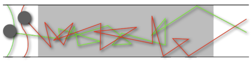
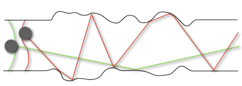
## Bulk vs surface disorder



Bulk disorder (mode-independent)

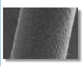
Mode number ≙ injection angle



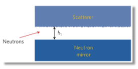
Surface disorder (mode-dependent)

**Coexistence of transport regimes [1]**

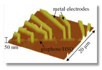
**Rough boundary systems relevant in many physical systems**



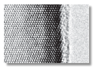
Optical fibers [2]



Gravitationally bound neutrons [3]

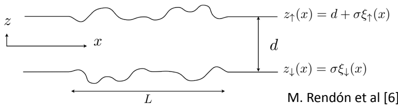


Graphene nanoribbons [4]



Silicon nanowires [5]

## Surface scattering theory (SST) [6]




M. Rendón et al [6]

$\sigma\xi(x), \sigma\xi'(x)$ → **Amplitude-gradient scattering (AGS)**

$$W(k) = \left\langle \left|\mathcal{F}_k\left[\xi(x)\right]\right|^2 \right\rangle$$

$\sigma^2\xi'(x)^2$ → **Square-gradient scattering (SGS)**

$$S(k) = \left\langle \left|\mathcal{F}_k\left[\xi'(x)^2\right]\right|^2 \right\rangle$$

- Transport from mode n to n′ determined by partial attenuation length

$$\frac{1}{L_{nn'}} = \frac{1}{L_{nn'}^{(AGS)}} + \frac{1}{L_{nn'}^{(SGS)}}$$

$$\frac{1}{L_{nn'}^{(AGS)}} = \sum_{m=n,n'} \frac{\sigma^2}{d^6}\frac{A_{nm}}{k_n k_m}\left[W(k_n+k_m) + W(k_n-k_m)\right]$$

$$\frac{1}{L_{nn'}^{(SGS)}} = \sum_{m=n,n'} \frac{\sigma^4}{d^4}\frac{B_{nm}}{k_n k_m}\left[S(k_n+k_m) + S(k_n-k_m)\right]$$

→ $A_{nn'}$, $B_{nn'}$ depend on wire symmetry

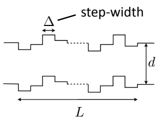
→ $k_n - k_m$: forward scattering

→ $k_n + k_m$: backward scattering

- Total attenuation length of nth mode

$$\frac{1}{L_n} = \sum_{n'} \frac{1}{L_{nn'}}$$

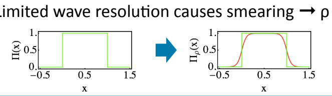
## Standard model: step-like wire [7]




Step boundary analytically modelled by 2N+1 smeared out steps $\Pi_\rho(x)$ featuring random heights $\alpha_n$:

$$\xi(x) = \sum_{n=-N}^{N} \alpha_n \Pi_\rho(x - n\Delta)$$

Limited wave resolution causes smearing → ρ



$$W(k_x) = \frac{1}{\Delta}\frac{4\pi^2\rho^2}{\sinh^2(\pi k_x \rho)}\sin^2(k_x\Delta/2)$$

$$S(k_x) = \frac{1}{\Delta}\frac{k_x^2\pi^2}{72}\frac{(1+k_x^2\rho^2)^2}{\sinh^2(\pi k_x\rho)}\Omega_N(k_x\Delta)$$

$$\Omega_N(x) = \left[\frac{4}{5}\left(1+\frac{1}{2N}\right)\left(7+2\cos(x)\right) + 2\left(1+\cos(x)\right)\frac{1}{2N}\frac{\sin^2\left[(N+1/2)x\right]}{\sin^2(x/2)}\right]$$

→ Resonance condition for specific parameter pairings:

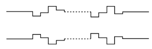
$$k_x\Delta = 2\pi M \quad \begin{cases} W(k_x) \to 0 \\ S(k_x) \propto N \end{cases}$$

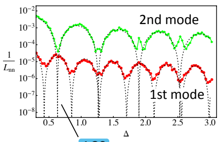
## Numerical results

- Two open modes
- Scan through different step-widths Δ
- Three symmetry classes:

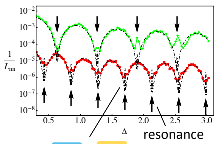
### Symmetric wire

$\xi_\uparrow = -\xi_\downarrow$ ➡





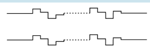

AGS

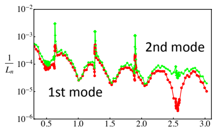


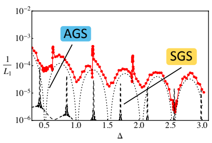

AGS & SGS resonance values

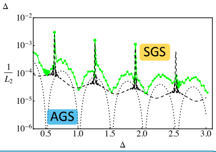
### Antisymmetric wire

$\xi_\uparrow = \xi_\downarrow$ ➡



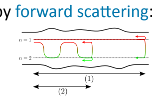



→ Standard SST [6] fails to reproduce behaviour of first mode

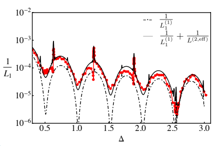
→ Explanation: backscattering increased due to mode mixing by forward scattering:

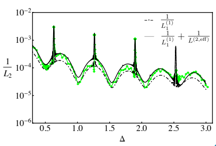


→ Corrections via effective higher order scattering contributions to the attenuation length

$$\frac{1}{L_n} = \frac{1}{L_n^{(1)}} + \frac{1}{L_n^{(2,\text{eff})}}$$

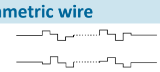
yield quantitative agreement:



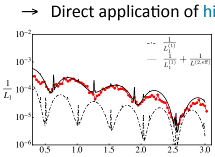


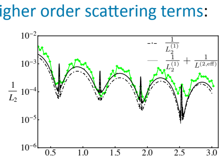
### Nonsymmetric wire

$\xi_\uparrow \neq \xi_\downarrow$ ➡



→ Direct application of higher order scattering terms:





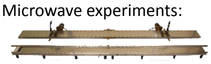
→ Quantitative agreement also in this case

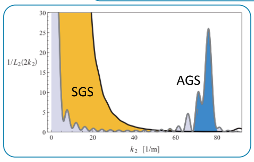
## Designing transmission bandgaps [8]

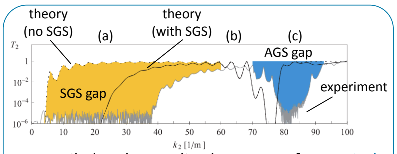
Transmission properties $W(k), S(k)$

→ Waveguide design:

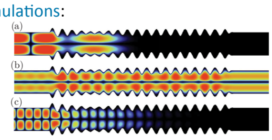
→ Microwave experiments:






→ Detailed understanding by means of numerical simulations:




## Summary

- Quantitative agreement between numerics and theoretically predicted new scattering mechanism
- Step-like wire geometry:
  → Pronounced backscattering peaks at resonant points $k_x\Delta = 2\pi M$
  → Effective higher-order scattering corrections improve agreement
- Waveguide design to fabricate predetermined transmission bandgaps
  → Successful demonstration in microwave experiments and numerical simulations

## References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