Route from spontaneous decay to complex multimode dynamics in cavity QED

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We study the non-Markovian quantum dynamics of an emitter inside an open multimode cavity, focusing on the case where the emitter is resonant with high-frequency cavity modes. Based on a Green's function technique suited for open photonic structures, we study the crossovers between three distinct regimes as the coupling strength is gradually increased: (i) overdamped decay with a time scale given by the Purcell modified decay rate, (ii) underdamped oscillations with a time scale given by the effective vacuum Rabi frequency, and (iii) pulsed revivals. The final multimode strong coupling regime (iii) gives rise to quantum revivals of the atomic inversion on a time scale associated with the cavity round-trip time. We show that the crucial parameter to capture the crossovers between these regimes is the nonlinear Lamb shift, accounted for exactly in our formalism.

Motivation

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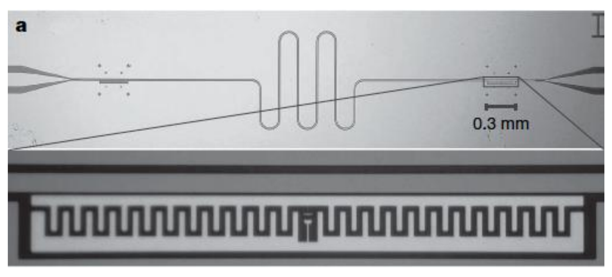
Quantum Dynamics of a two-level system (TLS) inside an complex open cavity: **One method for all regimes**

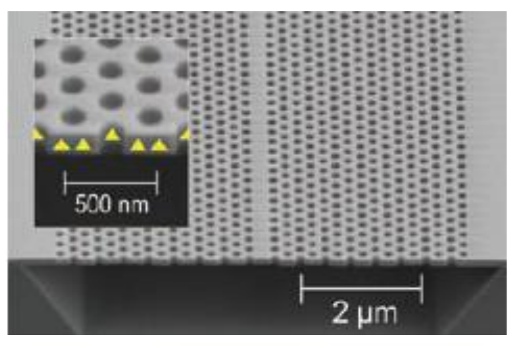
Circuit QED setups: qubit coupled to a microwave cavity

Fink et al.
Nature 454, 315 (2008)



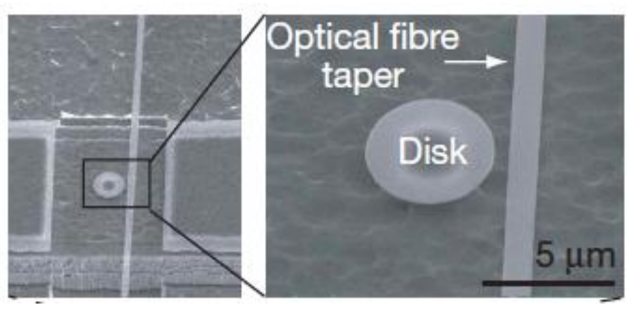
Cavity QED based on a quantum dot coupled to disordered photonic crystal waveguide

Sapienza et al.
Science, 1352 (2010)



Cavity QED based on strongly coupled microdisk-quantum dot system

Srinivasan & Painter
Nature 450, 862 (2007)



Laplace transformation & graphical analysis

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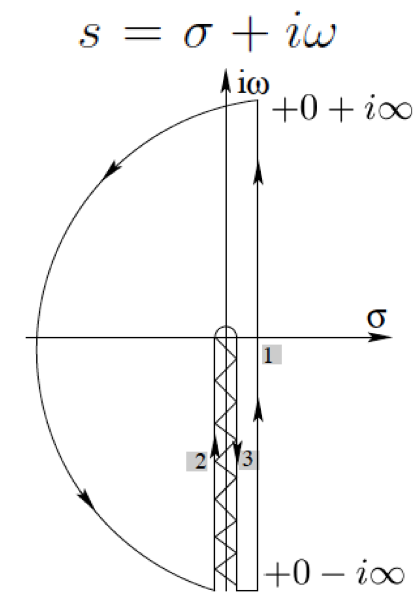
Laplace transformation: $\tilde{c}(s) = \int_0^\infty dt\, e^{-st} c(t)$

$$\tilde{c}(s+i\omega_a) = \frac{1}{s+i\omega_a + \frac{\gamma}{\pi}\int_0^\infty d\omega \frac{F(\omega)}{s+i\omega}}$$

$s = \sigma + i\omega$

Inverse Laplace transformation: $c(t) = \frac{e^{i\omega_a t}}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{st}\tilde{c}(s+i\omega_a)ds$

Contour completion:



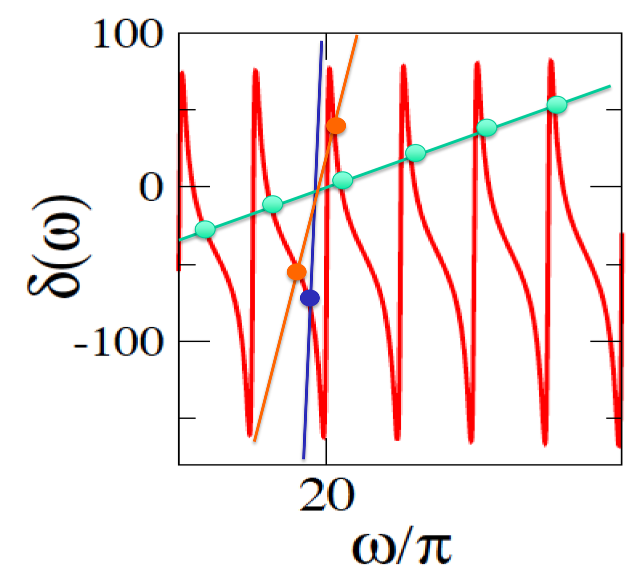
Solution for the amplitude of the upper level of TLS [2]: $c(t) = \frac{\gamma}{\pi} e^{i\omega_a t}\int_0^\infty d\omega\, U(\omega)\, e^{-i\omega t}$

Kernel function: $U(\omega) = \lim_{\varepsilon\to 0+} \frac{F(\omega)}{[\omega-\omega_a-\gamma\delta(\omega)]^2 + [\gamma F(\omega)+\varepsilon]^2}$

Nonlinear Lamb shift: $\delta(\omega) = \frac{1}{\pi}\mathcal{P}\int_0^\infty d\tilde{\omega}\frac{F(\tilde{\omega})}{\omega-\tilde{\omega}}$

Necessary condition for the resonances of U(ω): $\frac{\omega_r - \omega_a}{\gamma} = \delta(\omega_r)$

At resonances: $U(\omega_r) = \frac{1}{\gamma^2 F(\omega_r)}$



Model

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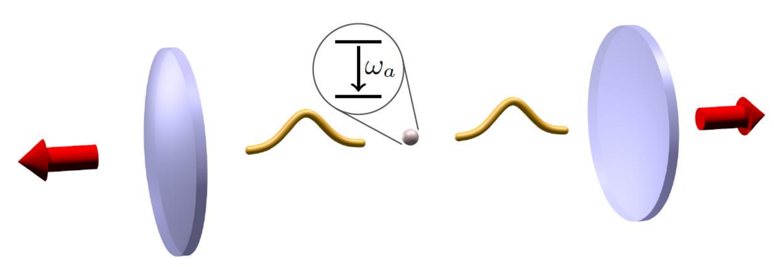
Two-level system with transition frequency ω_a inside an open cavity



Hamiltonian:

$$\mathcal{H} = \sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger a_\lambda + \frac{\hbar\omega_a}{2}\cdot\sigma_z + \hbar\sqrt{\frac{\gamma}{\pi}}\cdot\sum_\lambda [g_\lambda a_\lambda \sigma_+ + g_\lambda^\star a_\lambda^\dagger \sigma_-]$$

$a_\lambda^\dagger, a_\lambda$: boson creation and annihilation operators of a photon in λth mode

$\sigma_+, \sigma_-, \sigma_z$: Pauli operators associated with TLS

g_λ : coupling amplitudes at $r=r_a$ (dipole interaction)

$\gamma = \mu^2/\varepsilon_0\hbar$: dimensionless coupling strength

Ansatz to the Schrödinger equation

$$|\Psi(t)\rangle = c(t)e^{-i\omega_a t/2}|u\rangle|0\rangle + \sum_\lambda c_\lambda(t)|l\rangle|1_\lambda\rangle e^{-i(\omega_\lambda-\omega_a/2)t}$$

Assumptions: RWA, number of excitation=1

c(t): amplitude of the excited state; c(t=0)=1

$c_\lambda(t)$: amplitude of a single photon in λth mode; $c_\lambda(t=0)=0$

Volterra equation: $\dot{c}(t) = -\frac{\gamma}{\pi}\int_0^t dt' \int_0^\infty d\omega F(\omega) e^{-i(\omega-\omega_a)(t-t')}c(t')$

Spectral function: $F(\omega) = \rho(\mathbf{r}_a,\omega)\cdot|g(\omega)|^2$

Density of states: $\rho(x_a,\omega) = \mathrm{Im}\, G(x_a,x_a,\omega)/\pi$

Example: TLA inside 1D cavity

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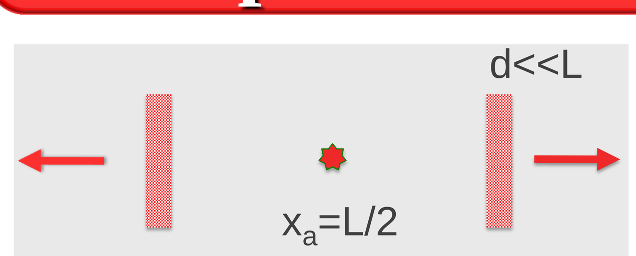
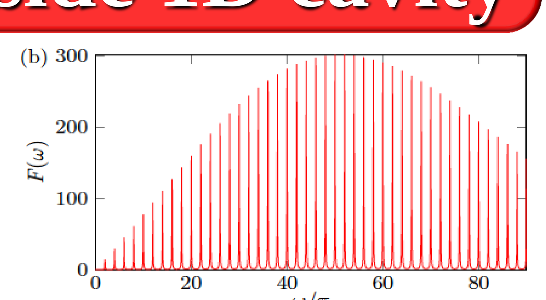


1D geometry



Spectral function

Eq. for the Green's function: $\left(\partial_x^2 + n^2\omega^2\right)G(x,x_a,\omega) = -\delta(x-x_a)$

Constant flux (CF) boundary conditions [1]:

$$\partial_x G(x=L,x_a,\omega) = i\omega G(x=L,x_a,\omega)$$
$$\partial_x G(x=0,x_a,\omega) = -i\omega G(x=0,x_a,\omega)$$

Spectral representation of the Green's function:

$$G(x,x',\omega) = -\sum_m \phi_m(x,\omega)\bar{\phi}_m^*(x',\omega)/[\omega^2-\omega_m^2(\omega)]$$

CF states with outgoing boundary conditions:

$$(\partial_x^2 + n^2\omega_m(\omega)^2)\phi_m(x) = 0$$
$$\partial_x\phi_m(x) = \pm i\omega\phi_m(x)$$

Biorthogonality condition: $\int_0^L dx\, n^2\bar{\phi}_m^*\phi_n = \delta_{mn}$

Results

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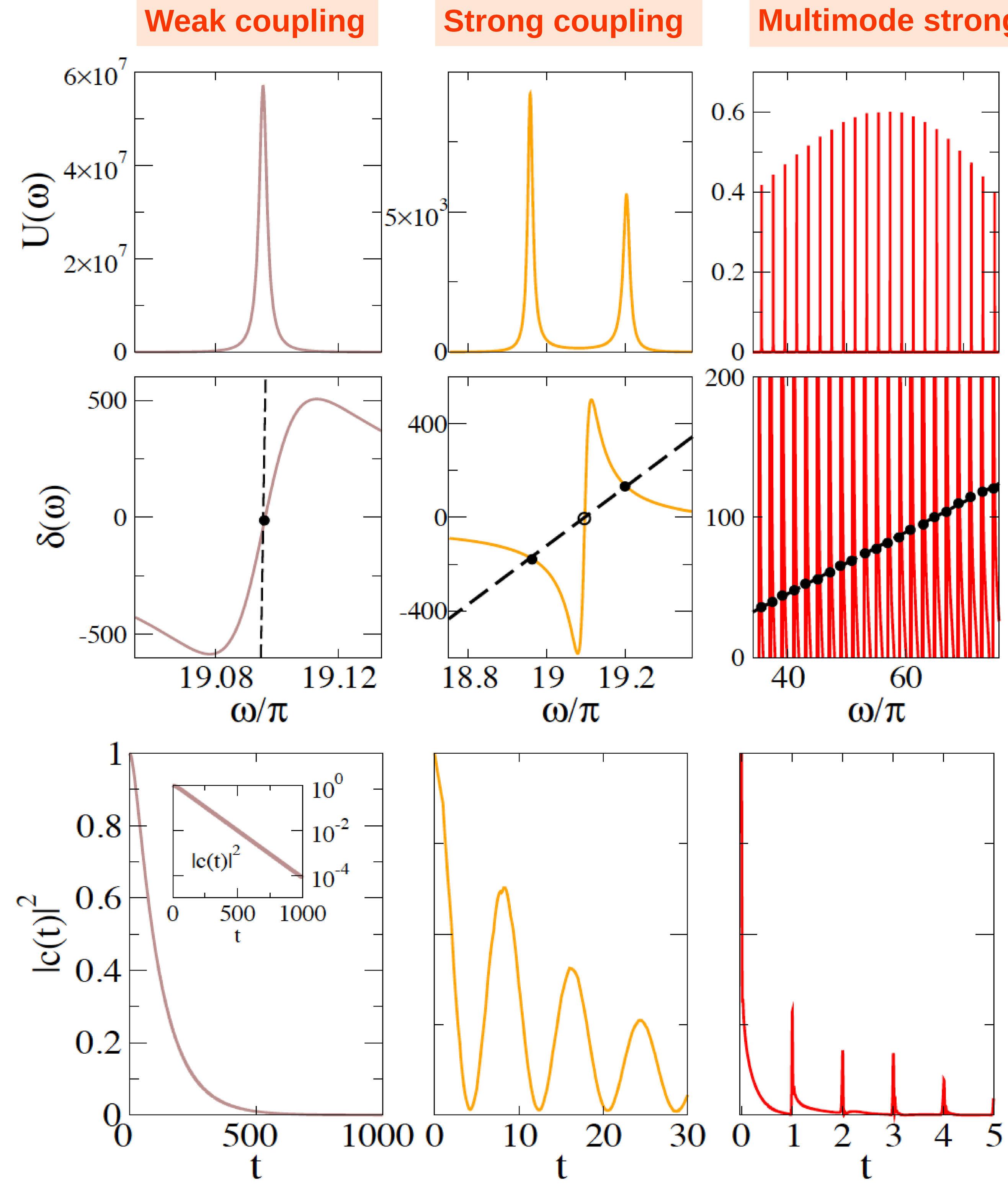


Kernel function & nonlinear shift for:

Left column: weak coupling regime for $\gamma=4\times10^{-6}$ with a single peak in U(ω) (Purcell modified spontaneous decay).

Middle column: strong coupling regime for $\gamma=2.5\times10^{-5}$ with a well-resolved Rabi splitting in U(ω) (regime of damped Rabi oscillations).

Right column: Multimode strong coupling regime for $\gamma=1.44\times10^{-3}$ with a multi-peak structure in U(ω) consisting of almost equidistant peaks (regime of revivals).

Temporal evolution of the excited state probability $|c(t)|^2$ of the TLS.

Multi-mode strong coupling regime featuring pulsed revivals at multiple integers of half the cavity round trip time.

t is normalized to half the cavity round-trip time L/c

Weak coupling regime — Single resonance at $\omega_r \approx \omega_a$ — Lorentzian $U(\omega) = \frac{F(\omega_a)}{[\omega-\omega_a-\gamma\delta(\omega_a)]^2+\gamma^2F(\omega_a)^2}$

Weisskopf-Wigner exponential decay of c(t)

Strong coupling regime — 3 intersections of nonlinear Lamb shift δ(ω) but 2 resonances (!) — Equation for the resonances: $\omega_r - \omega_a - \gamma\delta(\omega) = 0$, $\omega_r \approx \omega_a \pm \sqrt{2\gamma\omega_a}$

Jaynes-Cummings energy split=> Rabi oscillations

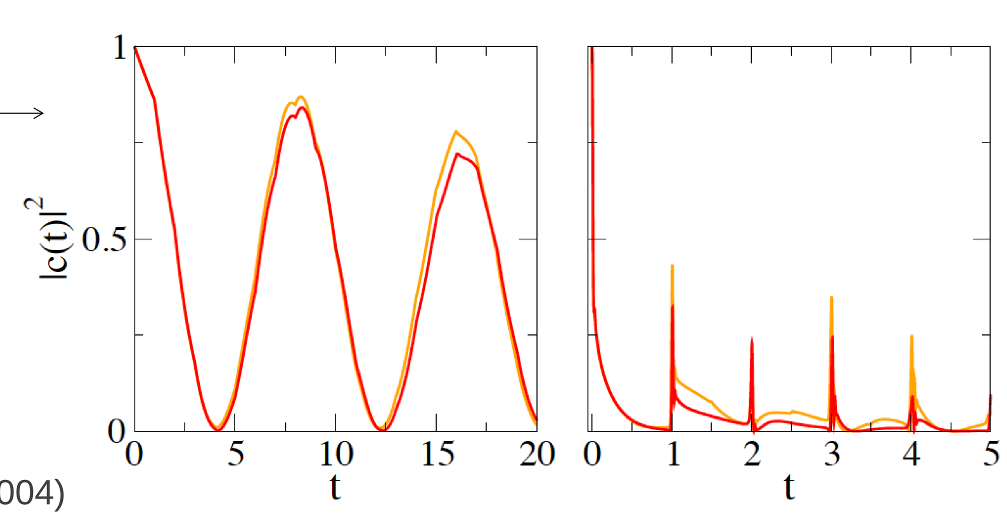
Multimode strong coupling regime — Many intersections=> coupling to large number of cavity modes. Every second intersection leads to a resonance

Periodic structure of U(ω) => sensitivity to a position of TLS

Regime of revivals

Comparison with rigorous QM formalism: system-and-bath Hamiltonian (Feshbach's projector technique) [3]

Set of Volterra equations for c(t) & $c_\lambda(t)$
Born+Markov approximations => single Volterra equation



References:

[1] H. E. Türeci, L. Ge, S. Rotter & A.D. Stone, Science 320, 643 (2008);
[2] D.O. Krimer, M. Liertzer, S. Rotter, &H. E. Türeci, arXiv:1306.4787 (2013)
[3] C. Viviescas & G. Hackenbroich, PRA 67, 013805 (2003); J.Opt. B 6, 211 (2004)