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Quasi-adiabatically Encircling Exceptional Points

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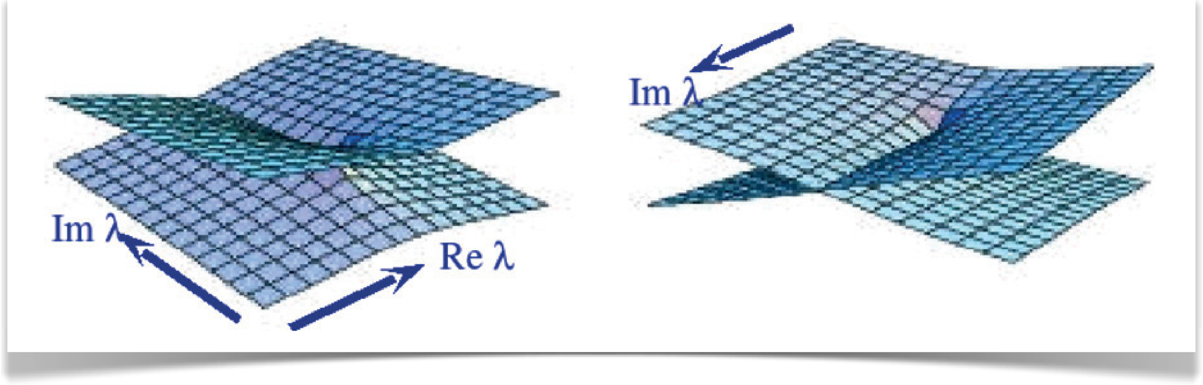
1. Introduction

• The quantum adiabatic theorem is a seminal result in the history of quantum mechanics [1].

Es besteht also bei unendlich langsamer Störung keine Wahrscheinlichkeit eines Quantensprunges.

• Recently, the applicability of adiabatic principles to non-Hermitian systems with gain or loss has attracted considerable interest. The spectrum of a non-Hermitian system is complex and may exhibit so-called exceptional points. Exceptional points are associated with phenomena that contradict our physical intuition, e.g., adiabatically encircling an exceptional point was predicted to effect a state-exchange [2].



• We present a detailed analytic study of quasi-adiabatically encircling exceptional points.

2. Model

• We consider the generic model of two coupled harmonic modes with gain or loss:

$$H=\begin{pmatrix}-\omega-i\gamma/2 & g\\ g & \omega+i\gamma/2\end{pmatrix},$$

$$\lambda_\mp=\mp\lambda=\mp\sqrt{(\omega+i\gamma/2)^2+g^2},$$

$$\vec r_-=\begin{pmatrix}\cos\frac{\vartheta}{2}\\ \sin\frac{\vartheta}{2}\end{pmatrix},\quad \vec r_+=\begin{pmatrix}-\sin\frac{\vartheta}{2}\\ \cos\frac{\vartheta}{2}\end{pmatrix},$$

$$\tan\vartheta=-g/(\omega+i\gamma/2).$$

• We assume that at least the oscillation frequency and the coupling, or the oscillation frequency and the decay can be controlled as a function of time.

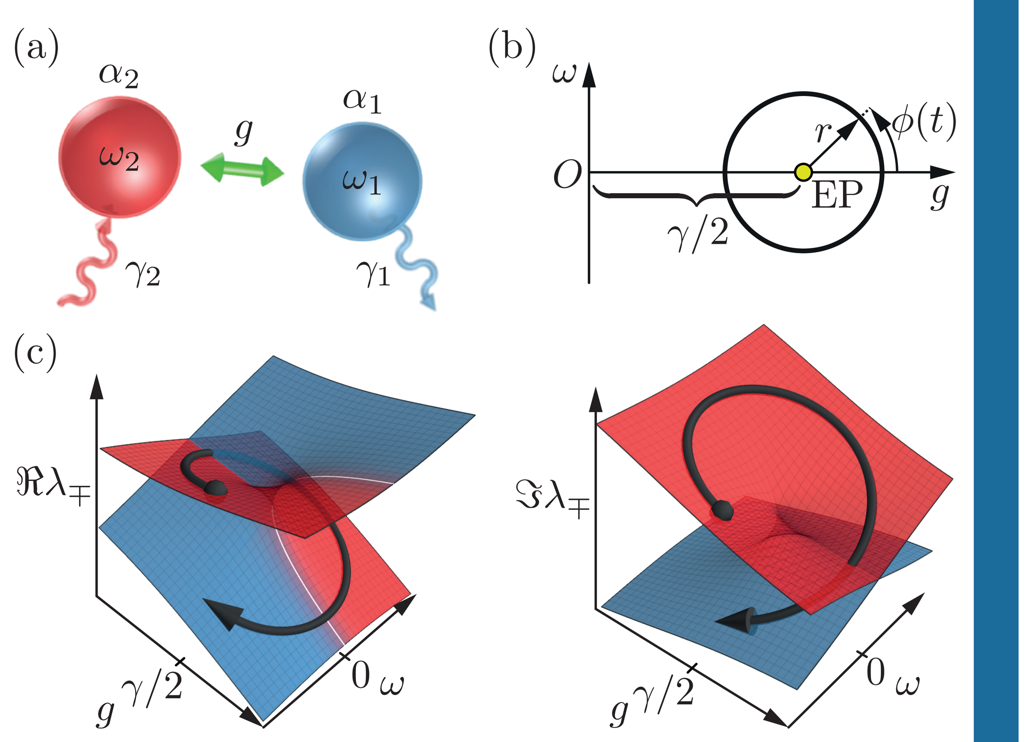


FIG. 1. (a) Cartoon of two coupled harmonic modes with gain or loss. (b) Example parametric path for fixed decay. (c) Real and imaginary parts of the spectrum in the vicinity of an exceptional point.

3. Numerical Examples

• Let us expand the state in the (instantaneous) eigenbasis:

$$(\alpha_1\quad\alpha_2)^T=c_-\vec r_-(t)+c_+\vec r_+(t).$$

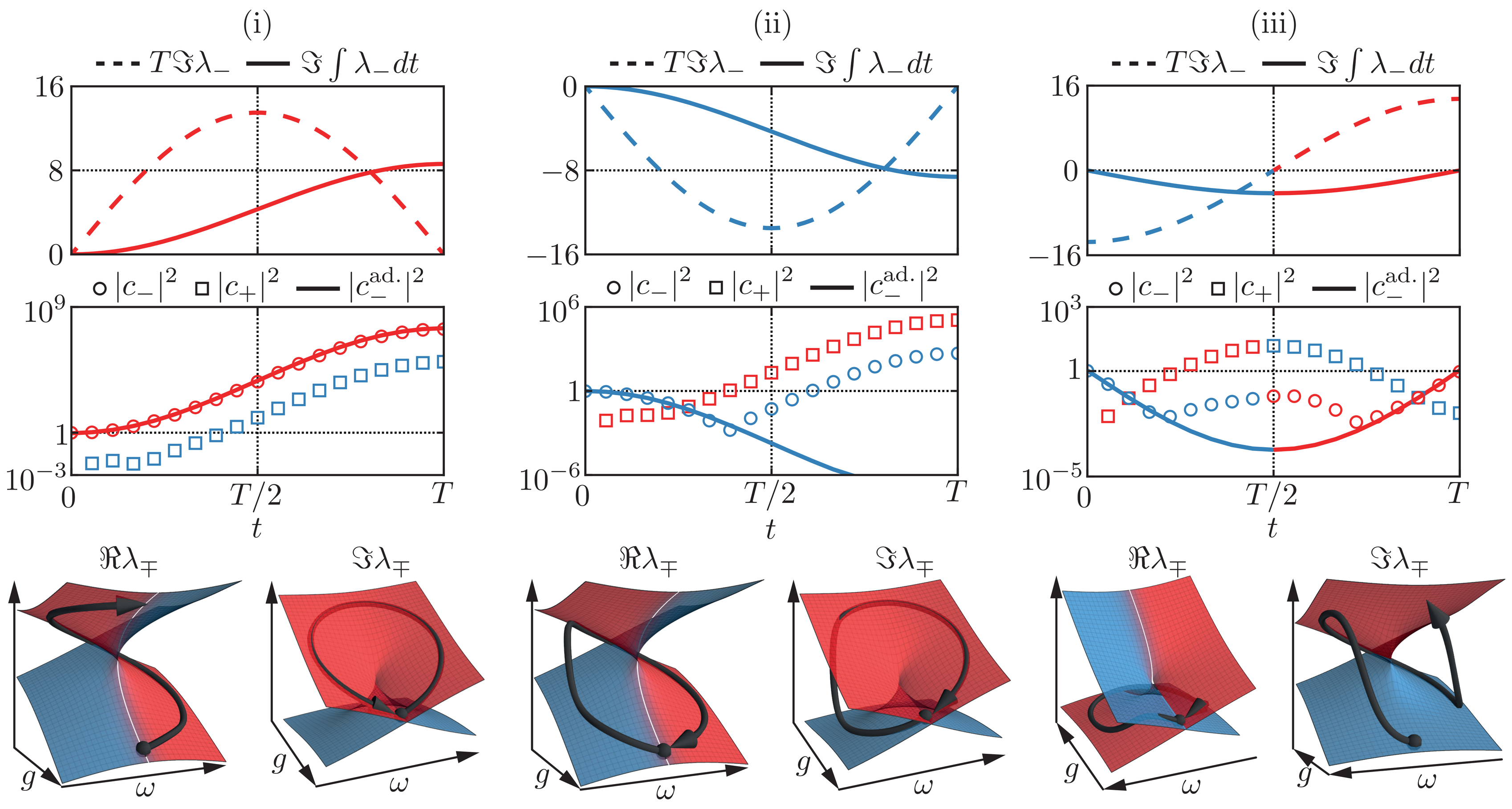


FIG. 2. (i, ii) Anticlockwise and clockwise encirclings. These examples exhibit chiral behaviour. (iii) An encircling for which the net gain vanishes. This example exhibits non-trivial slaving.

4. Theoretical Description

• In order to develop a general dynamical description we consider the evolution operator (in the eigenbasis):

$$\dot{\mathcal U}=-i\begin{pmatrix}-\lambda(t) & -f(t)\\ f(t) & \lambda(t)\end{pmatrix}\mathcal U,\quad \mathcal U=\begin{pmatrix}\mathcal U_{-,-} & \mathcal U_{-,+}\\ \mathcal U_{+,-} & \mathcal U_{+,+}\end{pmatrix},\quad \mathcal U(t=0)=1,$$

$$f(t)=\frac{[\omega(t)+\gamma(t)/2]\dot g(t)-g(t)[\dot\omega(t)+i\dot\gamma(t)/2]}{2i\lambda^2(t)}.$$

• Adiabaticity usually requires that the non-adiabatic coupling be much smaller than the 'distance' between eigenvectors:

$$\varepsilon(t)=\left|\frac{f(t)}{2\lambda(t)}\right|\ll 1.$$

• Even for an arbitrarily small yet non-vanishing non-adiabatic coupling the actual solution is significantly non-adiabatic (this is evident in Fig. 2). The non-adiabatic coupling is a *singular perturbation*.

• In order to describe the non-adiabatic character of the evolution operator we focus on the following relative non-adiabatic transition amplitudes [3]:

$$R_-(t):=\frac{\mathcal U_{+,-}(t)}{\mathcal U_{-,-}(t)},\quad R_+(t):=\frac{\mathcal U_{-,+}(t)}{\mathcal U_{+,+}(t)}.$$

• These transition amplitudes are solutions to a Riccati equation [4]:

$$\dot R_\mp=\mp 2i\lambda(t)R_\mp\mp if(t)(1+R_\mp^2).$$

5. Results

• Prohibition of simultaneous adiabatic behaviour in both eigenvectors [3]:

$$\lim_{t\to\infty}R_-(t)R_+(t)=1.$$

• Separation of time-scales [4]:

$$\underbrace{\dot R_\mp=\mp 2i\lambda R_\mp\mp if(1+R_\mp^2)}_{\text{fast time-scale}},$$

$$\underbrace{0=\mp 2i\lambda(t)R_\mp\mp if(t)(1+R_\mp^2)}_{\text{slow time-scale}}.$$

• Quasi-stationary fixed points (slow time-scale solution manifolds) [4]:

$$R^{\text{ad.}}_\mp(t)\simeq-\frac{f(t)}{2\lambda(t)}\ \text{ if }\ \mp\Im\lambda(t)>0,$$

$$R^{\text{n.ad.}}_\mp(t)\simeq-\frac{2\lambda(t)}{f(t)}\ \text{ if }\ \mp\Im\lambda(t)<0.$$

• Dynamical bifurcations [4]:

$$t=t_*\qquad \Im\lambda(t_*)=0,\ \Im\dot\lambda(t_*)\neq 0.$$

• Stability loss delay [5].

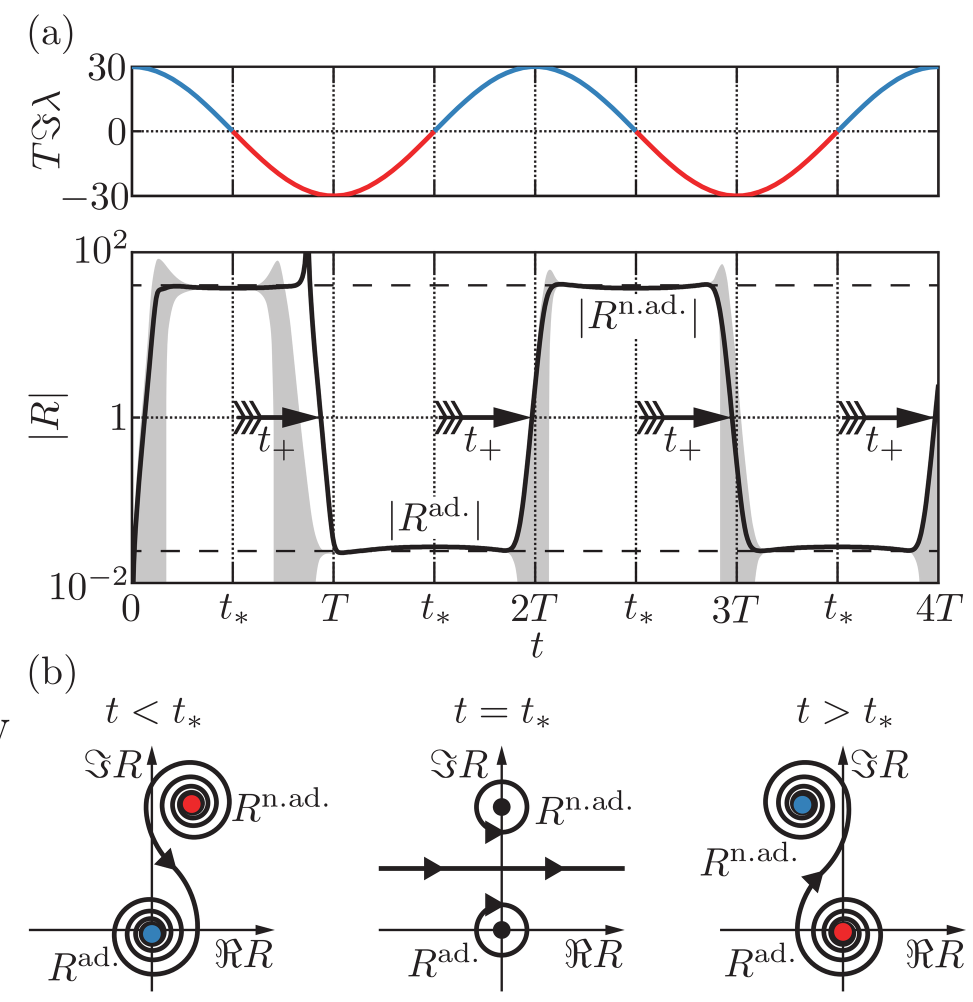


FIG. 3. (a) A generic solution for $R=:R$. The arrows denote delay times. The upper and lower horizontal dashed grid lines denote the fixed points. The shaded area denotes one standard deviation about the mean for 10,000 stochastic solutions. (b) Cartoons of the phase portrait near a bifurcation. Arrows denote the direction of time-evolution along an integral curve.

• Stokes phenomenon of asymptotics [6].

• Stability to noise [4].

• Unique long-term relaxation oscillation as a universal signature of quasi-adiabatically encircling exceptional points [4].

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SFB FoQuS Foundations and Applications of Quantum Science
FWF Der Wissenschaftsfonds.
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