Nonclassical light in the quantum dynamics of mesoscopic spin ensemble cavity systems

TIME-ADAPTIVE VARIATIONAL RENORMALIZATION GROUP METHOD FOR DISSIPATIVE SPIN CAVITY SYSTEMS

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Mesoscopic spin ensembles coupled to a cavity offer the exciting prospect of observing complex nonclassical phenomena with features intermediate between single spin and macroscopic spin ensemble cavity systems. Here, we demonstrate how the collective interactions in an ensemble of as many as 100 spins can be harnessed to obtain a periodic pulse train of nonclassical light. To unravel the full quantum dynamics and photon statistics, we develop a time-adaptive variational renormalization group method that accurately captures the underlying Lindbladian dynamics of the mesoscopic spin-cavity system.

Microscopic vs. macroscopic regime in hybrid quantum system

Hybrid quantum systems that allow the interaction of spins or emitters with modes of an electromagnetic field are the cornerstone of modern quantum technology

Macroscopic spin ensembles:
Large number of spins or emitters interacting with an EM field

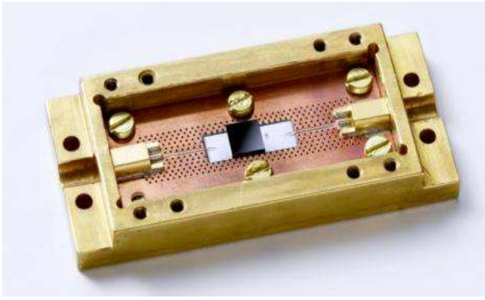


Image from TU Wien

10^{12} NV centers in microwave rsonator

NV centres, atomic gases, solid state rare-earth material

The light-matter interaction produces remarkable **collective behaviour**
But dynamics is essentially semiclassical

Quantum memories, computation, superradiance, spectral engineering…

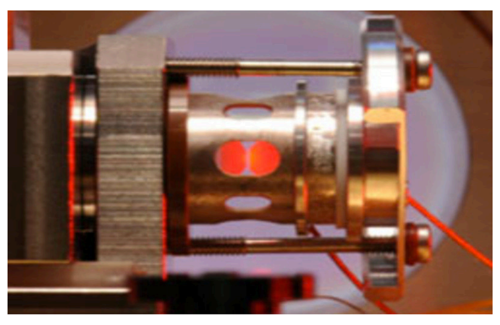


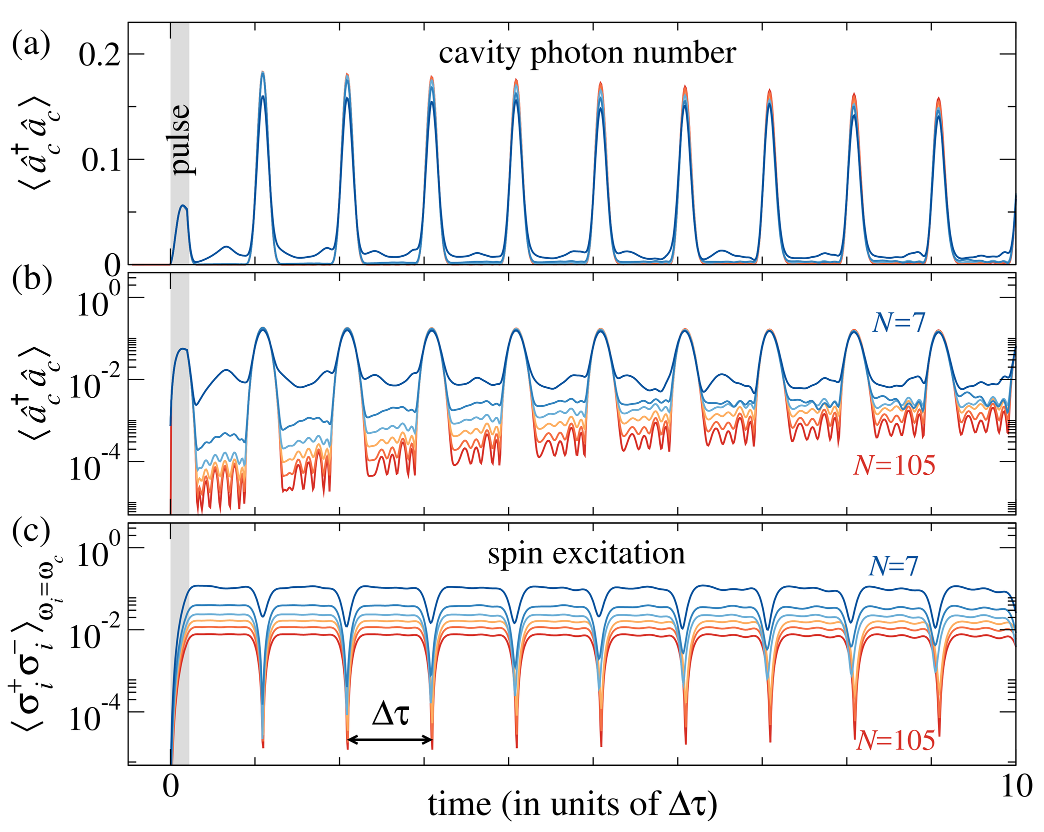
Image from http://www.mpq.mpg.de/quantumdynamics.

Microscopic spin system:
Single or few spins coupled to a quantum cavity

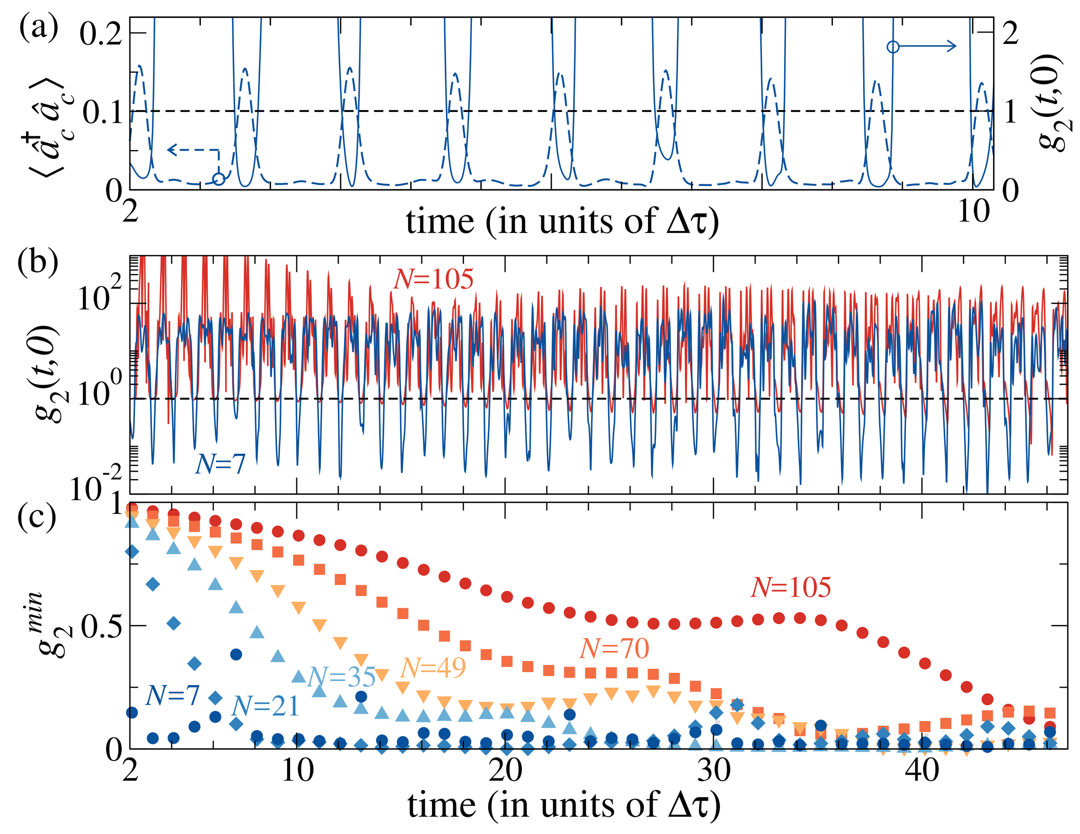
The spin-cavity interaction produces **exotic nonclassical phenomena**, but theoretically limited to few spins

Photon blockade, single photon source, antibunching, nonclassical light, entanglement…

Periodic pulses of nonclassical light from mesoscopic spin ensemble



Temporal evolution of the mesoscopic spin-cavity system. The figures show the cavity photon number in (a) linear and (b) log scale, and (c) spin excitation at resonance, varying with time, and for ensembles with N = 7 to 105 spins, shown with color varying from blue to red. The shaded region indicates the rectangular driving pulse and the interval between the periodic revivals is shown.

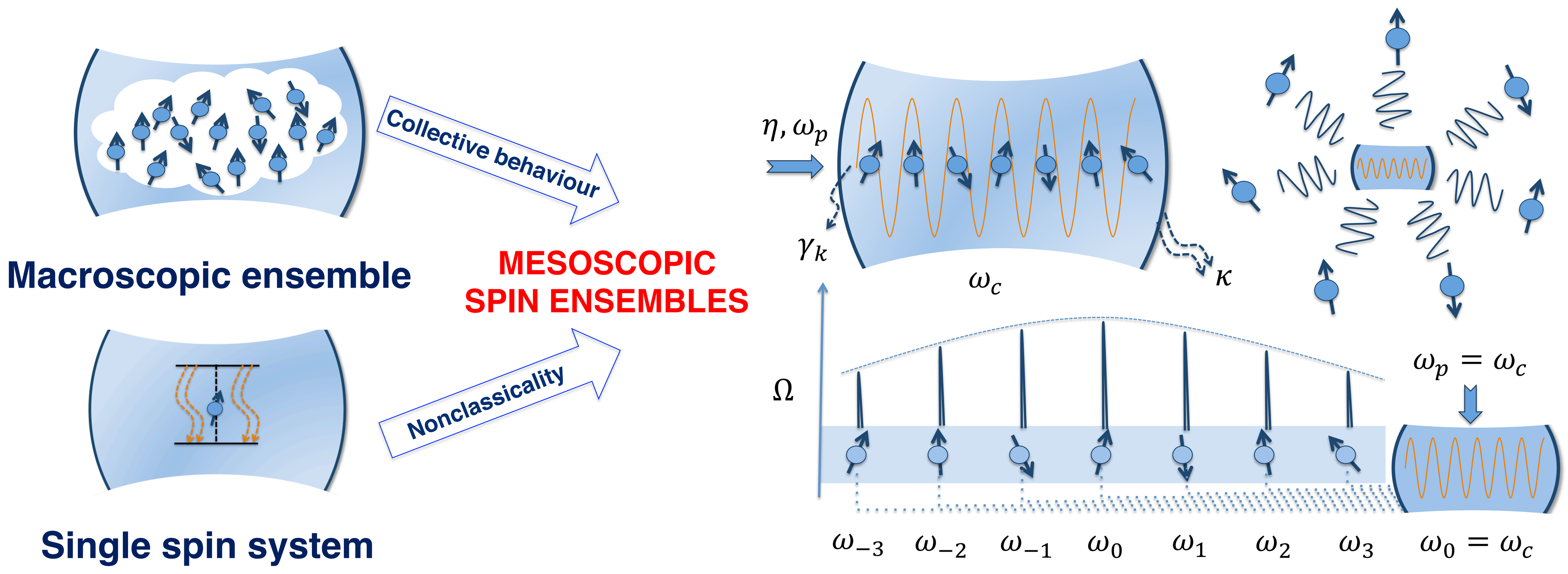


Photon statistics of the transient cavity field. (a) Equal-time second order correlation function in comparison with the cavity photon number, for N = 7. (b)Time evolution of $g_2(t,0)$ for N = 7 and 105. (c) Minimum value of $g_2(t, 0)$ close to a pulse revival, g_2^{min}, for N = 7 to 105. The horizontal black-dashed line is for $g_2(t,0)$ = 1 for coherent light.

The **collective properties** of the ensemble are highlighted in the periodic revival of light in the cavity. The **nonclassicality** is observed in the sub-Poissionian photon statistics of the antibunched light, as observed by the close to zero values of the second order correlation function. Thus, the mesoscopic pools the properties of both micro- and macroscopic regimes.

Collective behavior with nonclassicality: Mescoscopic regime

The largely uncharted mesoscopic regime provides the framework for combining the collective properties of the macroscopic regime with the innate nonclassicality of microscopic regime



Model: A mescoscopic ensemble of spins inside a quantum cavity with transition frequencies arranged in a **spectral frequency comb**.
Solving mesoscopic systems for different parameter regimes is limited due to the exponentially large Hilbert space. Need some powerful approach to tackle the open system dynamics.

Time-adpative variational renormalization group method for Lindbladian dynamics in spin-cavity system

The Tavis-Cummings Hamiltonian:

$$\mathcal{H} = \frac{1}{2}\sum_{k=1}^{N} \omega'_k \sigma_k^z + \omega_c \hat{a}_c^\dagger \hat{a}_c + i\sum_{k=1}^{N} g_k\left(\sigma_k^+ \hat{a}_c - \sigma_k^- \hat{a}_c^\dagger\right) + i\left(\eta(t)\hat{a}_c^\dagger e^{-i\omega_p t} - \eta^*(t)\hat{a}_c e^{i\omega_p t}\right)$$

The Lindblad master equation:

$$\frac{d\rho}{dt} = \mathcal{L}[\rho] = -i[\mathcal{H},\rho] + \kappa\left(\hat{a}_c\rho\hat{a}_c^\dagger - \frac{1}{2}\{\hat{a}_c\hat{a}_c^\dagger,\rho\}\right) + \sum_{k=1}^{N}\gamma_k\left(\sigma_k^-\rho\sigma_k^+ - \frac{1}{2}\{\sigma_k^-\sigma_k^+,\rho\}\right)$$

Superoperator space

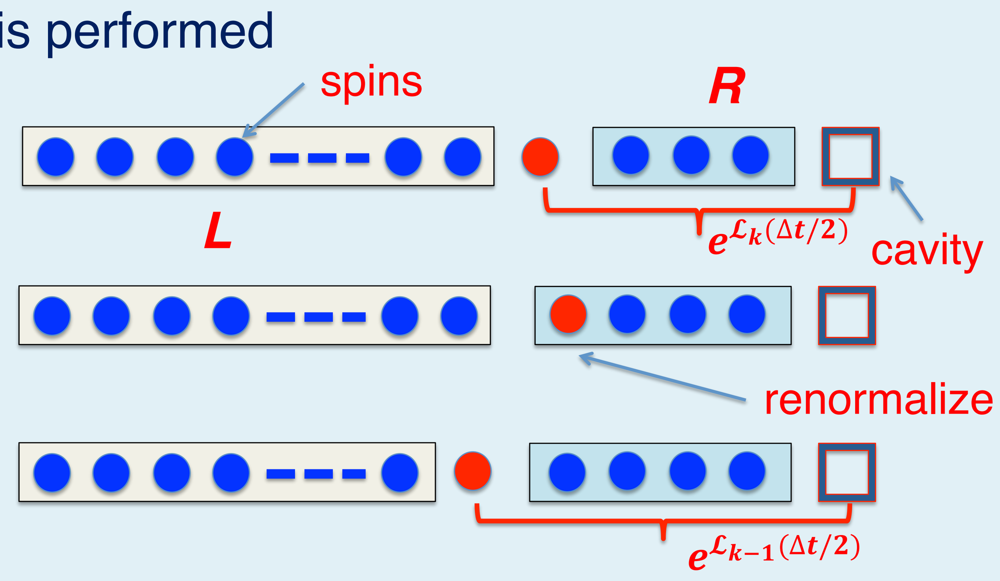
$$\rho = \sum_{i,j} p_{i,j}|i\rangle\langle j| \to \mathrm{vec}(\rho) \Rightarrow |\rho\rangle = \sum_{i,j} p_{i,j}|i,j\rangle$$

$$\hat{O}\rho = (\hat{O}\otimes\mathbb{I}_d)|\rho\rangle;\ \rho\hat{O} = (\mathbb{I}_d\otimes\hat{O}^{\mathrm{T}})|\rho\rangle;\ |\dot{\rho}\rangle = \mathcal{L}|\rho\rangle$$

$$\mathcal{L} = -i(\mathcal{H}\otimes\mathbb{I} - \mathbb{I}\otimes\mathcal{H}^{\mathrm{T}}) + \kappa\,\tilde{\mathcal{L}}_{\hat{a}_c} + \sum_k \gamma_k\,\tilde{\mathcal{L}}_{\sigma_k^-}$$

$$\tilde{\mathcal{L}}_{\hat{x}} = \hat{x}\otimes\hat{x}^* - \frac{1}{2}\hat{x}^\dagger\hat{x}\otimes\mathbb{I} - \frac{1}{2}\mathbb{I}\otimes\hat{x}^{\mathrm{T}}\hat{x}^*$$

Using ST and time-adaptive renormalization, a double sweep similar to *t*-DMRG or TEBD is performed



Central body system and evolution

$$\frac{d|\rho\rangle}{dt} = \mathcal{L}|\rho\rangle = \sum_{i=1}^{N}\mathcal{L}_i|\rho\rangle;\ \mathcal{L}_i = \mathcal{L}_{k,\gamma} + \frac{1}{N}\mathcal{L}_c$$

All spins are arranged around the central cavity and the interaction can be written as *N* individual spin-cavity (k) terms.

$$|\rho(t+\Delta t)\rangle = e^{\mathcal{L}\Delta t}|\rho(t)\rangle = e^{\sum\mathcal{L}_i\Delta t}|\rho(t)\rangle = \mathbb{V}(\Delta t)|\rho(t)\rangle$$

Suzuki-Trotter (ST) decomposition

$$\mathbb{V}(\Delta t) = e^{\sum\mathcal{L}_i\Delta t} = e^{\mathcal{L}_N\left(\frac{\Delta t}{2}\right)}e^{\mathcal{L}_{N-1}\left(\frac{\Delta t}{2}\right)}\ldots e^{\mathcal{L}_1(\Delta t)} \times \ldots e^{\mathcal{L}_{N-1}\left(\frac{\Delta t}{2}\right)}e^{\mathcal{L}_N\left(\frac{\Delta t}{2}\right)} + \mathcal{O}(\Delta t^3)$$

Schmidt decomposition and renormalization of the reduced superoperator space

$$|\rho\rangle = \sum_{\tilde{k}=1}^{S}\alpha_{\tilde{k}}|\tilde{k}_L\rangle\otimes|\tilde{k}_R\rangle \approx \sum_{\tilde{k}=1}^{D}\alpha_{\tilde{k}}|\tilde{k}_L\rangle\otimes|\tilde{k}_R\rangle$$

$$\mathcal{R}_{L(R)} = \mathrm{Tr}_{R(L)}(|\rho\rangle\langle\rho|);\ 1 \le S \le \min[\mathrm{rank}(\mathcal{R}_L,\mathcal{R}_R)]$$

The Schmidt rank (S) and coefficient $\alpha_{\tilde{k}}$ is a measure of the total correlations in the system and is typically much smaller than the rank of the reduced superoperator. The system can be thus be renormalized to a much smaller dimension in the superoperator space, $S << D$, as done in traditional variational numerical renormalization and tensor-network methods. This is performed during the steps in the Lindblad dynamics.

In conclusion, we demonstrate that mesoscopic ensembles of spins coupled to a quantum cavity provide an interesting new platform for studying and tailoring non-classical light fields. Based on recent experiments, implementing the proposed comb-shaped ensemble should be readily possible and an attractive option for creating a pulsed quantum source of light.

H.S. Dhar, M. Zens, D.O. Krimer, and S. Rotter, *Variational Renormalization Group for Dissipative Spin-Cavity Systems: Periodic Pulses of Nonclassical Photons from Mesoscopic Spin Ensembles*, **arXiv:1806.02394**

Acknowledgments

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