Periodic cavity state revivals from atomic frequency combs – Supplemental Material

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(Dated: September 6, 2021)

ENERGY SPECTRUM

In this supplementary section, we show how the strong spin-cavity coupling distorts the eigenvalue spectrum of the uniformly coupled frequency comb. Introducing collective spin operators for each subensemble $J^z_\mu = \frac{1}{2}\sum_{k=1}^{N'} \sigma^z_k$ and $J^\pm_\mu = \sum_{k=1}^{N'} \sigma^\pm_k$, the Tavis-Hamiltonian given in Eq.(1) of the main text can be rewritten as

$$H = \omega_c a^\dagger a + \sum_\mu \omega_\mu J^z_\mu + g_\mu \sum_\mu (J^+_\mu a + J^-_\mu a^\dagger), \qquad (1)$$

where $\mu = \{-(m-1)/2, ..., (m-1)/2\}$, $m = 7$ is the number of subensembles, and $N' = N/m$ is the number of spins in each subensemble. In the following, spins inside the same frequency subensemble are described in the collective spin basis $|J_\mu, m_\mu\rangle = |N'/2, -N'/2 + q_\mu\rangle \equiv |q_\mu\rangle$, where q_μ is the number of excitations in the μ-th subensemble. With the use of

$$J^z_\mu |J_\mu, m_\mu\rangle = m_\mu |J_\mu, m_\mu\rangle = (-N'/2 + q_\mu) |q_\mu\rangle, \qquad (2)$$

and

$$\begin{aligned} J^\pm_\mu |J_\mu, m_\mu\rangle &= \sqrt{J_\mu(J_\mu+1) - m_\mu(m_\mu \pm 1)}\, |J_\mu, m_\mu\rangle \\ &= \sqrt{N'/2 + N'q_\mu - q_\mu^2 \pm (N'/2 - q_\mu)}\, |q_\mu\rangle, \end{aligned} \qquad (3)$$

we can set up the single- and two-excitation subspace for the coupled spin-cavity system. We write the $1+m$ basis states of the single-excitation subspace of the combined spin-cavity system as $|1_c\rangle |0_\mu\rangle$ and $|0_c\rangle |1_\mu\rangle$. Here, $|1_c\rangle |0_\mu\rangle$ denotes a state with a single excitation in the cavity part and no excitations in the spin ensemble, whereas $|0_c\rangle |1_\mu\rangle$ denotes m states where the single excitation is in the μ-th subensemble (with all other subensembles unexcited) and no excitations in the cavity. The action of the Hamiltonian (1) in this single-excitation basis is the given by

$$H |1_c\rangle |0_\mu\rangle = \left(\omega_c - \frac{N'}{2}\omega_\Sigma\right) |1_c\rangle |0_\mu\rangle + \sum_\mu \Omega_\mu |0_c\rangle |1_\mu\rangle, \qquad (4)$$

$$H |0_c\rangle |1_\mu\rangle = \left(\omega_\mu - \frac{N'}{2}\omega_\Sigma\right) |0_c\rangle |1_\mu\rangle + \Omega_\mu |1_c\rangle |0_\mu\rangle, \qquad (5)$$

with $\omega_\Sigma = \sum_\mu \omega_\mu$.

Using the same notation as above, the basis states of the two-excitation subspace can be written as $|2_c\rangle |0_\mu\rangle$, $|1_c\rangle |1_\mu\rangle$, $|0_c\rangle |1_\mu\rangle |1_\nu\rangle$, $|0_c\rangle |2_\mu\rangle$ (in total $1 + m + m\,(m-1)/2 + m$ basis states). The Hamiltonian (1) acting on these states gives

$$\begin{aligned} H |2_c\rangle |0_\mu\rangle = &\left(2\omega_c - \frac{N'}{2}\omega_\Sigma\right) |2_c\rangle |0_\mu\rangle \\ &+ \sqrt{2}\sum_\mu \Omega_\mu |1_c\rangle |1_\mu\rangle, \end{aligned} \qquad (6)$$

$$\begin{aligned} H |1_c\rangle |1_\mu\rangle = &\left(\omega_c + \omega_\mu - \frac{N'}{2}\omega_\Sigma\right) |1_c\rangle |1_\mu\rangle \\ &+ \Omega_\mu \sqrt{2}\, |2_c\rangle |0_\mu\rangle + \sum_{\nu\neq\mu} \Omega_\nu |0_c\rangle |1_\mu\rangle |1_\nu\rangle \\ &+ \Omega_\mu \sqrt{2 - 2/N'}\, |0_c\rangle |2_\mu\rangle, \end{aligned} \qquad (7)$$

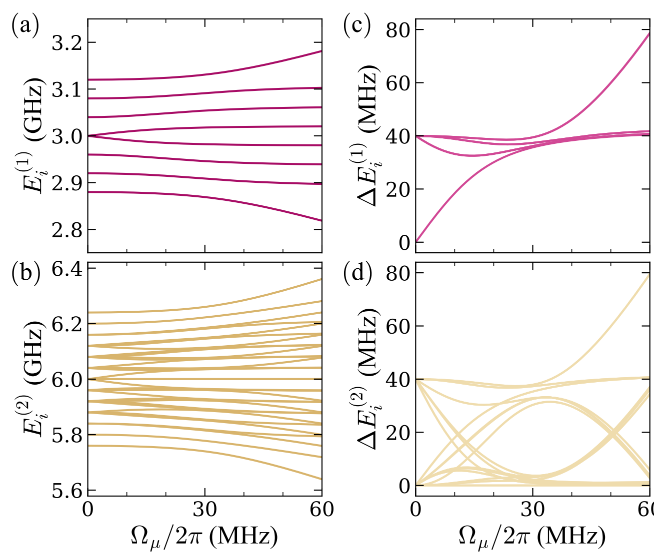

FIG. 1. (a,b) Energy spectrum of the singe- and two-excitation subspace of the Tavis-Cummings Hamiltonian (1) as a function of the collective coupling strength Ω_μ. Here we consider the simplest case of an equidistant atomic frequency comb structure with $\Delta\omega/2\pi = 40$ MHz uniformly coupled to a single cavity mode. (c,d) Corresponding spacings $\Delta E_i^{(1/2)}$ of neighboring energy levels. The spin-cavity coupling lifts the equidistant energy spectrum of the atomic frequency comb yielding a non-equidistant spectrum for the compound spin-cavity system.

$$H\left|0_c\right\rangle\left|1_\mu\right\rangle\left|1_\nu\right\rangle = \left(\omega_\mu + \omega_\nu - \frac{N'}{2}\omega_\Sigma\right)\left|0_c\right\rangle\left|1_\mu\right\rangle\left|1_\nu\right\rangle + \Omega_\mu\left|1_c\right\rangle\left|1_\nu\right\rangle + \Omega_\nu\left|1_c\right\rangle\left|1_\mu\right\rangle \tag{8}$$

$$H\left|0_c\right\rangle\left|2_\mu\right\rangle = \left(2\omega_\mu - \frac{N'}{2}\omega_\Sigma\right)\left|0_c\right\rangle\left|2_\mu\right\rangle + \Omega_\mu\sqrt{2-2/N'}\left|1_c\right\rangle\left|1_\mu\right\rangle . \tag{9}$$

With the above equations one can set up the Tavis-Cummings Hamiltonian in the single- and two-excitation basis and solve its eigenvalues numerically. The resulting energy spectrum is presented in Fig. 1 as a function of the coupling strength Ω_μ for the simple case of an equidistant frequency comb with $\Delta\omega/2\pi = 40$ MHz and uniform coupling. The strong coupling leads to a normal-mode splitting, which lifts the degeneracy of the cavity mode and the central (resonant) spins. Consequently, the first rung of the energy ladder in the strong coupling regime consists of $m+1$ levels instead of m in the uncoupled case. The coupling between the cavity and spin ensemble thereby shifts the energy levels of the spin-cavity system such that they are no longer equidistant for strong coupling Ω_μ. In the main text, we show how this drawback of the strong-coupling regime can be overcome by adjusting the individual coupling strengths of the frequency comb.